Vulnerability assessment of monumental masonry structures including uncertainty

Savvas Saloustros[1], Luca Pelà[2], Francesca Romana Contrafatto[3], Pere Roca[4] and Ioannis Petromichelakis[5]

Abstract. Existing heritage structures are frequently composed of diverse masonry typologies, corresponding either to various structural members (e.g. arches, walls, piers) or to additions constructed in different eras. The identification of the material properties of the different masonry typologies is usually demanding due to the high cost of the necessary specialized in-situ experimental testing procedures and to the restrictions posed by the cultural value of historical buildings. This lack of information underlines the importance of probabilistic studies considering the uncertainties connected with the evaluation of the material properties. Such activities become essential in studies dealing with the conservation of built cultural heritage against hazardous events, such as earthquakes. This work investigates the seismic vulnerability assessment of large monumental structures with complex geometry. The church of Santa Maria del Mar in Barcelona is considered as a case study, and a representative macro-element of the bay structure is studied against in-plane horizontal loading through pushover analysis. A Monte Carlo simulation is used to estimate the effect of the uncertainties on the material properties, which are considered as random variables. The developed fragility curves express the safety level and the damage expected on the structure for different seismic hazard scenarios.

Keywords: **e**arthquake, nonlinear static analysis, churches, uncertainty, Monte Carlo simulation, fragility curves.

1 Introduction

Unreinforced masonry buildings represent a significant amount of the built heritage in earthquake-prone regions. Past and recent seismic events have demonstrated the low capacity of these structures against horizontal loading, resulting in important losses in terms of human lives and cultural heritage. The gravity of all these losses suggests an urgent need for evaluating the seismic safety of these structures and plan proper interventions, if necessary.

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[1] Postdoctoral Researcher, Department of Civil and Environmental Engineering, Universitat Politècnica de Catalunya (UPC-BarcelonaTech), Barcelona, Spain, savvas.saloustros@upc.edu

[2] Associate Professor, Department of Civil and Environmental Engineering, Universitat Politècnica de Catalunya (UPC-BarcelonaTech), Barcelona, Spain, luca.pela@upc.edu

[3] MSc in Structural Analysis of Monuments and Historical Constructions, Universitat Politècnica de Catalunya (UPC-BarcelonaTech), Barcelona, Spain, contrafattofrancesca@tiscali.it

[4] Full Professor, Department of Civil and Environmental Engineering, Universitat Politècnica de Catalunya (UPC-BarcelonaTech), Barcelona, Spain, pere.roca.fabregar@upc.edu

[5] PhD Candidate, Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, USA, ip2315@columbia.edu

Assessing the vulnerability of existing buildings is a very demanding task involving a high amount of uncertainty in the characterization of their geometry and materials. The study of masonry buildings is often compromised by the high number of uncertainties caused by the variety of structural typologies, the heterogeneity of the materials, the complex morphology of the structural members and the important transformations that, in many cases, they experience during their life. Moreover, in the case of historical constructions, the necessary inspection and experimental activities are often severely limited by their outstanding cultural value.

This paper deals with the seismic vulnerability of complex monumental buildings including the analysis of the uncertainties related to the material properties. The church of Santa Maria del Mar in Barcelona is selected as a representative case study. A two-dimensional Finite Element (FE) model of a typical bay structure of the church has been studied under transversal seismic loads. The uncertain material properties have been considered as random variables. The effect of these uncertainties have been included by analysing a sample of possible material combinations for the same structure, generated by means of a Monte Carlo Simulation. The capacity of each model has been assessed by running 200 non-linear static analyses. Four limit states have been considered for damage grade assessment and the seismic demand has been evaluated by using the N2 method. Finally, analytical fragility curves have been derived, defining the probability of occurrence of each damage grade for different seismic scenarios.

2 Methodology for the seismic assessment of monumental structures

2.1 Numerical model

The seismic behaviour of a representative bay (Fig. 1) of the church of Santa Maria del Mar in Barcelona is studied under transversal horizontal loading. A plane-stress FE model has been prepared based on the 3D FE model presented in [1], such that the two models show equivalent response in terms of stiffness and capacity under both gravitational and horizontal in-plane loading proportional to the mass distribution, following the procedure reported in [2, 3]. The structure has been discretized by using 37780 triangular constant strain finite elements and 17749 nodes. The material model adopted for the simulation of the non-linear response of masonry is the rotating smeared cracking model available in the finite element code DIANA-FEA [4]. A parabolic stress-strain relationship has been chosen to simulate the compressive behaviour, while the model presents an exponential softening in tension. These stress-strain relationships are regularized according to the crack-bandwidth approach.

The seismic performance of the studied macro-element has been evaluated by means of a non-linear static (pushover) analysis performed in two steps. The first one includes the application of the vertical gravitational loads and the second one the seismic horizontal load proportional to the mass. A regular Newton Raphson method with an arc-length strategy has been used to solve the corresponding nonlinear equations at each step of the analysis. Both force and displacement criteria have been used to check convergence at each analysis step with a tolerance of 0.01. The performed analyses include the effect of both material and geometrical non-linearity.

2.2 Uncertainty of the material parameters

The mechanical parameters necessary for the structural analysis are commonly determined by in-situ or laboratory tests. These data are uncertain due to the natural variation of mechanical properties (i.e. aleatoric uncertainty) and the impossibility to achieve a complete knowledge of their variation within the structure (i.e. epistemic uncertainty). This work considers the uncertainty related to three mechanical parameters of masonry, namely the tensile strength, the compressive strength and the elastic modulus.

According to available studies on the church, four categories of structural members with similar mechanical properties can be distinguished, see Fig. 1. Among them, the vaults and the single-leaf walls have been considered as a reference material, and the mechanical properties of the rest are defined as a function of the reference ones. Six parameters have been considered and modelled as random variables. Three of them are mechanical properties of the reference material, i.e. the compressive strength f_c, the tensile strength f_t and the elastic modulus E. These parameters possess both aleatoric and epistemic uncertainties and can vary according to a log-normal probability density function, in agreement with [5, 6]. Table 1 presents the mean (μ), log-normal mean (μ_{ln}) and standard deviation (σ_{ln}), which have been chosen following the suggestions in [5]. Fig. 2 presents the distribution of f_c, f_t and E for the reference material, with the mean and standard deviation values of Table 1.

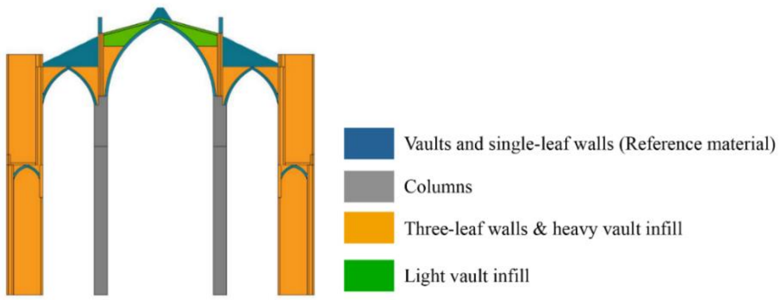


Figure 1. Categorization of the structural members in the representative bay of the church of Santa Maria del Mar.

Table 1 Probability distributions and parameters for the reference material.

Random variable	Distribution	Mean μ	Log-normal mean μ_{ln}	Standard deviation σ_{ln}
f_c	lognormal	7.00 [MPa]	1.94	0.05
f_t	lognormal	0.26 [MPa]	-1.37	0.22
E	lognormal	2900 [MPa]	7.96	0.13

Table 2 Ranges of variation for the coefficients C_c, W_c and I_c.

Random variable	Distribution	min	max
C_c	uniform	0.7	0.8
W_c	uniform	1.1	1.3
I_c	uniform	0.17	0.23

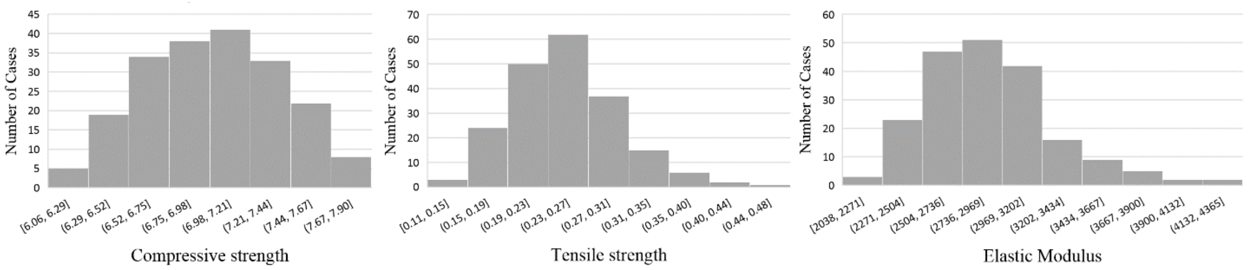


Figure 2. Distribution of compressive strength, tensile strength and elastic modulus for the reference material.

The mechanical properties of the rest of the materials of the structure have been defined to be proportional to the reference ones, with coefficients of proportionality, C_c for the columns, W_c for the three-leaf walls and heavy infill of the vaults and I_c for the light infill of the vaults. These coefficients are assumed to possess only epistemic uncertainties and, for this reason, uniform distributions have been adopted to define the corresponding random variables. Table 2 presents the ranges of variation for these coefficients. Note that these values have been selected such that the possible material parameters are in agreement with the suggestions of [7]. Finally, the tensile and compressive fracture energy have been defined as a function of the compressive strength f_c as $G_f^t = 0.025(f_c/10)^{0.7}$ and $G_f^c = df_c$ according to [8, 9], with the ductility index d=1.6 *mm*. For more details on the selection of the random variables see [10].

Based on the aforementioned assumptions, and for comparison purposes as will be shown in Section 3.1, a reference model has been developed by defining the mechanical properties according to the mean values of table C8A.2.1 of the Appendix C8A of [7], which are presented in Table 3.

Table 3 Mechanical properties used in the reference model

Structural Element	f_c [MPa]	f_t [MPa]	E [MPa]	G_f^t $[J/m^2]$	$G_f^c [J/m^2]$
Vaults and single-leaf walls	7.00	0.26	2900	19.5	11200
Columns	8.50	0.33	3590	22.3	13600
Three-leaf walls & heavy infill vaults	5.30	0.20	2230	16.0	8480
Light infill vaults	1.43	0.06	613	6.4	2288

2.3 Methodology for uncertainty analysis

2.3.1 Uncertainty analysis

The effect of the material uncertainty on the seismic response is evaluated through a Monte Carlo Simulation (MCS). By means of MCS, $N = 200$ different numerical models for the same structure have been generated within the input space R^n, where $n = 6$ is the number of the assumed input random variables. Each random variable is distributed according to the selected probability distribution and interval of variation, see Section 2.2. Subsequently, a mapping model (Fig. 3) maps each sample to the results' space $Z \subseteq R^m$, where m is the number of the result variables that need to be evaluated. In this work the m variables are the peak ground accelerations for each of the four defined limit states (i.e. $m = 4$), which represent the seismic demand, see Section 2.3.2. The methodology of this study is schematized in Fig 3 and is based on the "method C" proposed in [5].

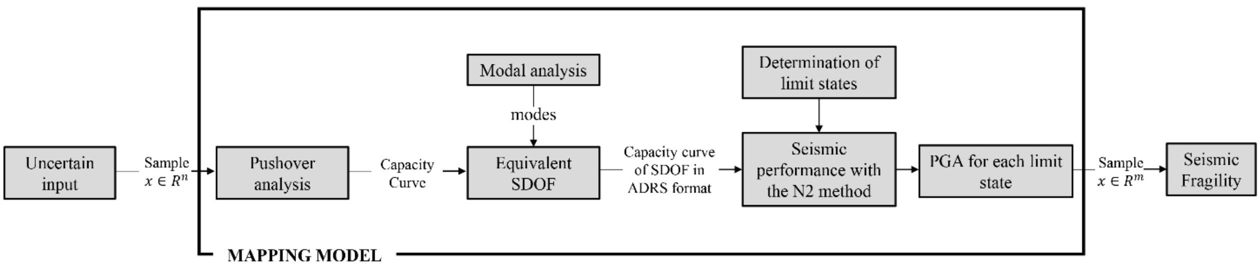


Figure 3 Scheme of the adopted methodology of vulnerability assessment including material uncertainty

2.3.2 Seismic demand

The seismic demand of the analysed structure has been defined considering four limit states according to the mechanical method proposed in [11]. Those limit states (Table 4) are a function of the yield (d_y) and ultimate displacements (d_u) of the idealized capacity curve corresponding to the equivalent single degree of freedom system. The latter idealized curve has been constructed according to [12], with the ultimate displacement identified in correspondence to a decrease of 20% of the maximum load capacity of the structure.
The acceleration spectrum used is the one proposed by the Eurocode 8 [12] considering the type of Soil D, according to the studies carried out on the foundation soil of the church [13]. The N2 method procedure [14] has been used to identify the peak ground acceleration (PGA) necessary to produce each damage limit state.

Table 4 Definition of the limit states

Limit State	1	2	3	4
Displacement	0.7 d_y	1.5 d_y	0.5 (d_y + d_u)	d_u
Damage level	Slight (LS1)	Moderate (LS2)	Extensive (LS3)	Complete (LS4)

3 Results

3.1 Capacity curves

Fig. 4a shows the 200 capacity curves in terms of spectral displacement (S_d) against spectral accelerations (S_a), as well as the 16%, 50% and 84% percentile curves. The last ones are curves representing the S_a level that is not exceeded by 16%, 50% and 84% of the individual capacity curves for every S_d value, making reference to a normal distribution of S_a [15]. It is easy to distinguish two groups of capacity curves with important differences in load and displacement capacities. The first group exhibits a horizontal acceleration capacity lower than 0.08g and a very brittle post-peak response. This low ductility is due to the collapse of the right buttress due to shear cracking, as shown in Fig. 5a. The percentile curves demonstrate that the cases resulting in this collapse mechanism are below 16% of the total analysed cases. The rest of the models predict a global collapse mechanism, characterized by cracking in the main and lateral naves and cracking at the right and left buttresses, as shown in Fig. 5b. These cases show higher capacity and ductility levels compared to the ones affected by the local collapse mechanism.
Fig. 4b presents the mean, median (50% percentile) capacity curves, together with the pushover curve of the reference case having the properties presented in Table 3. The higher position of the median curve compared to the mean, implies that for each displacement the capacity curves below the median are located slightly farther from the median than the capacity curves above it. This is more evident in Fig. 4a, where the curves below the median appear significantly more dispersed than the curves above, especially in terms of spectral acceleration. Consequently, for the adopted distributions of the uncertain parameters, for the given S_d the distribution of S_a is unsymmetrical and shifted to the higher values. The capacity curve obtained with the reference model is much higher than the mean curve and the median one. This implies that samples with one or more of the input parameters below the mean push the capacity curve downwards than samples above it push it upwards. Consequently,

the use of a single numerical model with deterministic mechanical properties, even though following the suggested values from the literature, would overestimate the structural capacity for the analysed case.

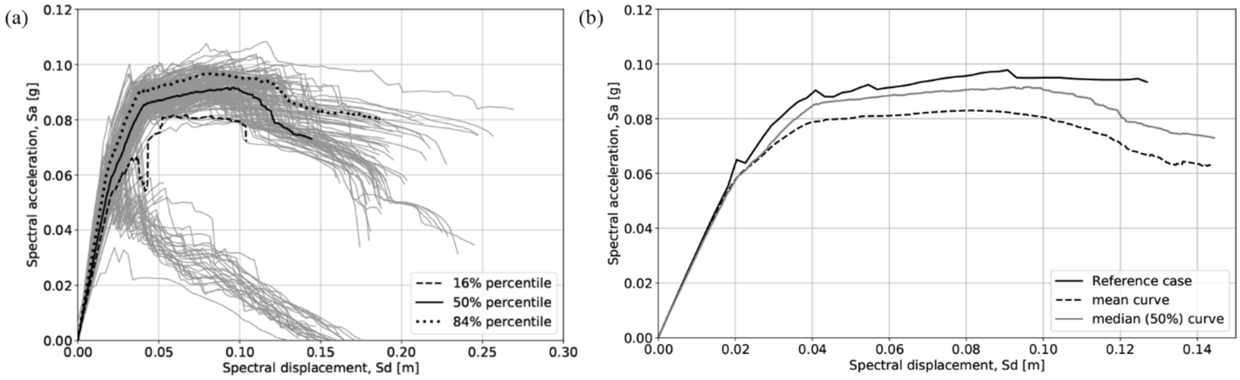


Figure 4. (a) The obtained capacity curves of the 200 performed analyses, (b) Mean, median and reference capacity curves

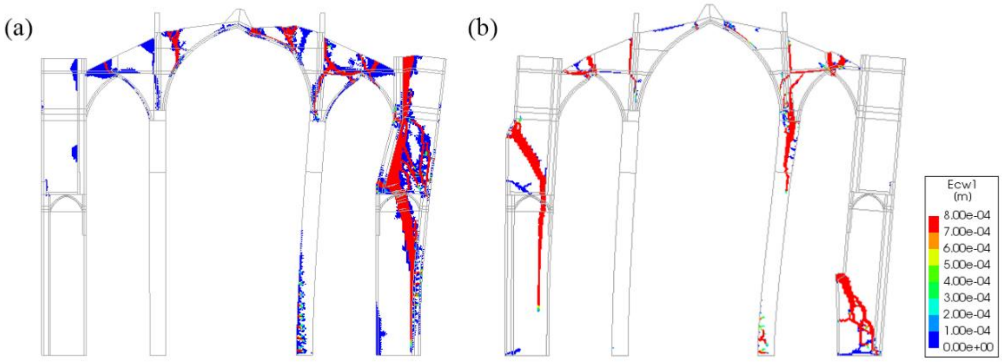


Figure 5. Crack widths at the end of the analysis in: (a) case with a collapse mechanism characterized by the shear failure of the right buttress, (b) case with a global collapse mechanism involving the main and lateral vaults and the two lateral buttresses of the structure.

3.2 Seismic fragility

The seismic fragility of the structure has been defined in terms of fragility curves, representing the probability that the structure will reach or overcome the considered limit state as a function of the PGA.

Fragility functions have been built according to the analytical formulation proposed in [16] in terms of PGA for each limit state, see Fig. 6. The vertical line for 0.04 g represents the expected PGA in the city of Barcelona.

Fig. 7 illustrates the probability of damage occurrence corresponding to each limit state. The expected seismic demand in Barcelona is sufficient to reach just the first limit state (LS1), since the corresponding probability of occurrence is equal to 100%. This limit state corresponds to important damage at the buttresses above the vaults of the lateral naves for both the possible collapse mechanisms, as shown in Figs. 8-9. The second limit state has a probability of occurrence equal to 50%, and is associated with more cracking at the vaults and buttresses at the lateral naves for all the studied cases, as well as important shear cracking for the cases resulting in the local shear failure of the right buttress (second column of Figs. 8-9). A noticeably low probability is reached for the third and the fourth limit states, equal respectively to 12% and 5%. The associated damage for these two limit states are presented in the third and fourth columns of Figs. 8-9. The results demonstrate that for the actual earthquake demand, the probability of occurrence of the collapse is very low.

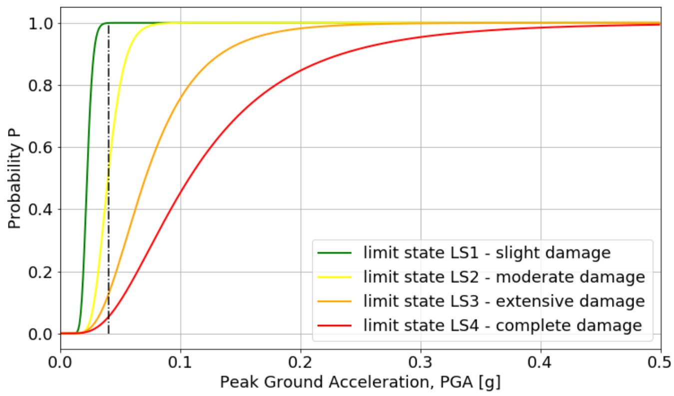


Figure 6 Fragility curves for each limit state in terms of PGA

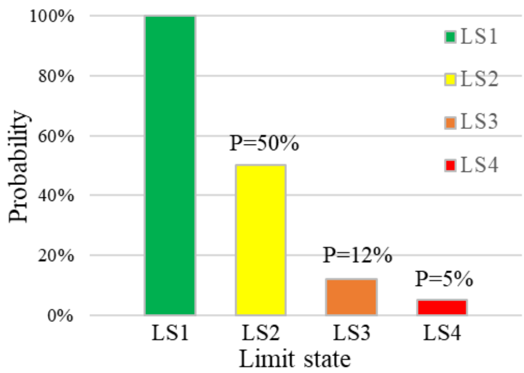
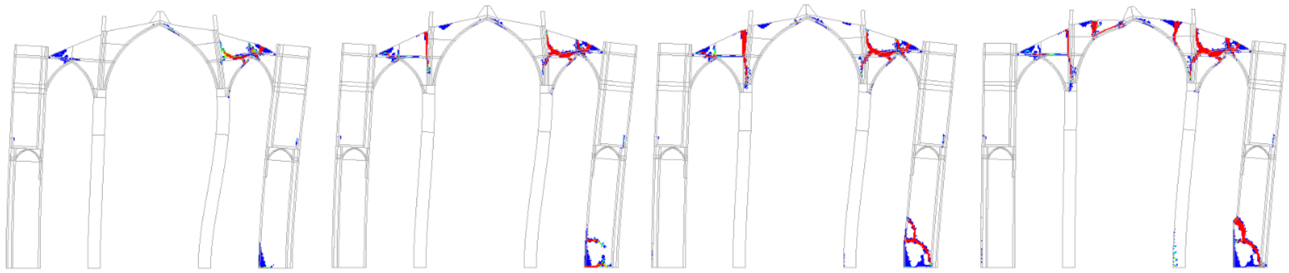
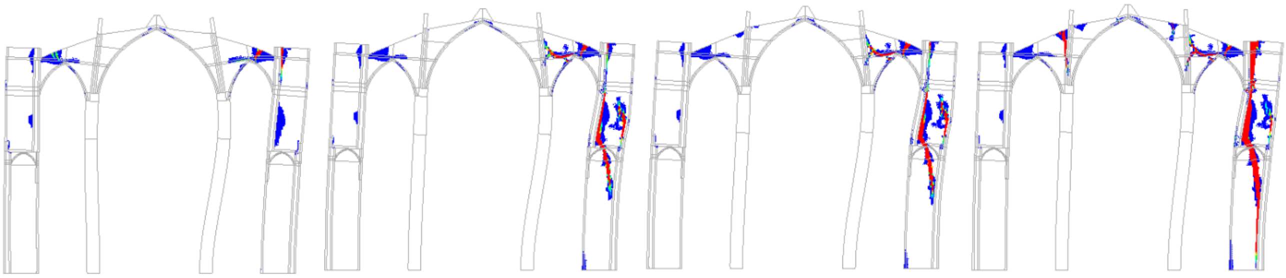


Figure 7 Probability of occurrence of each limit state for PGA=0.04g



P[LS1|PGA=0.04g] = 100% P[LS2|PGA=0.04g] = 50% P[LS3|PGA=0.04g] = 12% P[LS4|PGA=0.04g] = 5%

Figure 8 Correspondence between limit states and cracking for an analysed case of a global collapse mechanism



P[LS1|PGA=0.04g]= 100% P[LS2|PGA=0.04g]= 50% P[LS3|PGA=0.04g]= 12% P[LS4|PGA=0.04g] = 5%

Figure 9 Correspondence between limit states and cracking for an analysed case of a local collapse not involving the left buttress

4 Conclusions

This paper presented a vulnerability assessment of the typical bay of Santa Maria del Mar church in Barcelona considering the uncertainties associated with the material properties. A plane-stress model has been constructed and studied against horizontal transversal loading. A set of uncertain material properties have been considered in the numerical analysis as random variables. The seismic demand has been defined in terms of four limit states, representing different damage grades in the structure. The adopted methodology predicts that two collapse mechanisms are possible for the analysed structure. The first is a local one, not involving one of the lateral buttresses and the second is a global collapse mechanism with damage in the main and lateral naves as well as cracking in the lateral buttresses. The obtained results, expressed in terms of fragility curves for the different limit states, show that for the seismic hazard of Barcelona the analysed tranversal bay of the church would present, in case of earthquake, important damage affecting the buttresses above the vaults of the lateral naves, but with low probability of collapse.

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