# Damage detection and imaging in solids based on recorded elastodynamic response

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### 1 Introduction

- 2 Source localization
- 3 Defect localization
- 4 Numerical examples

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## Detection and Localization of Damage

- $\bullet$  Usually based on response recordings at a number of sensors to monitor structural integrity  $^1$
- Detection : comparison of recordings to a reference (undamaged) state
- Localization : Inverse Problem usually ill-posed
- Solution : Time-Reversal (TR) computational tool introduced by Fink et. al.<sup>2</sup>
- Achieves refocusing of the wave on the source
- Sending back the recorded signals but reversed in time
- Two step approach
  - Forward step
  - Backward step

<sup>1.</sup> GE Stavroulakis, (2000) Inverse and crack identification problems in engineering mechanics 2. Fink et. al., (2000) Time-reversed acoustics

- TR is a physical process
- It exploits the time reversibility (based on spatial reciprocity and time reversal invariance) of linear wave equations
- Robust and Simple technique for source localization
- Has been applied in Acoustics<sup>3</sup>, Elastodynamics<sup>4</sup>, Electromagnetism, Hydrodynamics etc.
- Finds several applications in medicine, telecommunications, underwater acoustics, seismology, engineering structures, etc.
- TR can be used for scatterer localization

example

<sup>3.</sup> L Borcea, G Papanicolaou, C Tsogka and J Berryman, (2002) Imaging and time reversal in random media

<sup>4.</sup> D Givoli, (2014) Time Reversal as a Computational Tool in Acoustics and Elastodynamics.

- Imaging techniques that exploit the fundamental idea of TR
- Description of the numerical implementation for the elastic wave propagation
- Utilization of the Green's function of the Elastodynamic equation to apply imaging techniques
- 2D rectangular bounded domain with elastic behavior
- Investigation of the influence of the boundaries in imaging
- Investigation of the main factors that affect the quality of the image

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- Simulated Numerically using a mixed finite element formulation <sup>5</sup>
- The domain contains one source at  $oldsymbol{x}_s$  and  $N_r$  receivers at  $oldsymbol{x}_r$ ,  $r=1,\ldots,N_r$
- Wave propagation model, Velocity Stress (first order)

$$\rho \frac{\partial \mathbf{v}}{\partial t} - \operatorname{div} \sigma = \delta(\mathbf{x} - \mathbf{x}_s) f(t) \mathbf{e}_i$$
$$A : \frac{\partial \sigma}{\partial t} - \dot{\varepsilon} = 0 \qquad \qquad \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \mathbf{v}_i}{\partial x_j} + \frac{\partial \mathbf{v}_j}{\partial x_i} \right)$$

- Homogeneous Neumann boundary conditions and zero initial conditions
- Excitation function f(t) is a Ricker pulse centered at a known  $t_0$
- $\bullet\,$  The response is being recorded during total time T

<sup>5.</sup> E Bécache, P Joly and C Tsogka (2002) A new family of mixed finite elements for the linear elastodynamic problem.

- Always performed numerically in SHM applications
- ullet The recorded signal is time reversed and retransmitted at  $x_r$
- Sensors acting as sources introducing right hand side loading terms

$$\rho \frac{\partial \mathbf{v}^{TR}}{\partial t} - \operatorname{div} \sigma^{TR} = \sum_{r=1}^{N_r} \delta(\boldsymbol{x} - \boldsymbol{x}_r) \mathbf{v}(\boldsymbol{x}_r, T - t)$$

- Homogeneous Neumann boundary conditions and zero initial conditions
- Refocusing at time  $t_{RF} = T t_0$

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• Solution of the backward problem

$$\mathbf{F}(\boldsymbol{x}_r,t) = \mathbf{v}(\boldsymbol{x}_r,T-t) \Leftrightarrow \hat{\mathbf{F}}(\boldsymbol{x}_r,\omega) = \overline{\hat{\mathbf{v}}(\boldsymbol{x}_r,\omega)} e^{i\omega T}$$

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$$\mathbf{F}(\boldsymbol{x}_r, t) = \mathbf{v}(\boldsymbol{x}_r, T - t) \Leftrightarrow \hat{\mathbf{F}}(\boldsymbol{x}_r, \omega) = \overline{\hat{\mathbf{v}}(\boldsymbol{x}_r, \omega)} e^{i\omega T}$$

• Solution of the backward problem

$$\begin{aligned} \mathbf{F}(\boldsymbol{x}_{r},t) &= \mathbf{v}(\boldsymbol{x}_{r},T-t) \Leftrightarrow \hat{\mathbf{F}}(\boldsymbol{x}_{r},\omega) = \overline{\hat{\mathbf{v}}(\boldsymbol{x}_{r},\omega)} e^{i\omega T} \\ \mathbf{v}^{TR}(\boldsymbol{x},t) &= \mathbf{G}(\boldsymbol{x},\boldsymbol{x}_{r},t) \star_{t} \mathbf{F}(\boldsymbol{x}_{r},t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\mathbf{G}}(\boldsymbol{x},\boldsymbol{x}_{r},\omega) \hat{\mathbf{F}}(\boldsymbol{x}_{r},\omega) \mathrm{d}\omega \end{aligned}$$

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• Evaluation of  $\mathrm{v}^{TR}({\pmb{x}},t)$  at the refocusing time  $T-t_0$ 

$$\mathbf{v}^{TR}(\boldsymbol{x}, t = T - t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\mathbf{G}}(\boldsymbol{x}, \boldsymbol{x}_r, \omega) \overline{\hat{\mathbf{v}}(\boldsymbol{x}_r, \omega)} e^{i\omega t_0} d\omega$$

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• Imaging functional - numerical approximation

$$I(\boldsymbol{x}) = \frac{1}{2\pi} \sum_{i,r} \hat{G}^{h}(\boldsymbol{x}, \boldsymbol{x}_{r}, \omega_{i}) \overline{\hat{v}(\boldsymbol{x}_{r}, \omega_{i})} \Delta \omega_{i}$$

## Numerical implementation of Imaging

• More precisely

$$\begin{split} \mathbf{I}(\boldsymbol{x}) &= \begin{bmatrix} \mathbf{I}_x(\boldsymbol{x}) \\ \mathbf{I}_y(\boldsymbol{x}) \end{bmatrix}, \quad \overline{\hat{\mathbf{v}}(\boldsymbol{x}_r, \omega_i)} = \begin{bmatrix} \overline{\mathbf{v}_x(\boldsymbol{x}_r, \omega_i)} \\ \overline{\mathbf{v}_y(\boldsymbol{x}_r, \omega_i)} \end{bmatrix}, \\ \hat{\mathbf{G}}^h(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) &= \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) & \mathbf{G}_{xy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \\ \mathbf{G}_{yx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) & \mathbf{G}_{yy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \end{bmatrix} \end{split}$$

ullet The final image I can be a combination (e.g. SRSS) of  $\mathbf{I}_x$  and  $\mathbf{I}_y$ , where

$$\begin{bmatrix} \mathbf{I}_x(\boldsymbol{x}) \\ \mathbf{I}_y(\boldsymbol{x}) \end{bmatrix} = \frac{\Delta\omega}{2\pi} \sum_{i,r} \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_x(\boldsymbol{x}_r, \omega_i)} + \mathbf{G}_{xy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_y(\boldsymbol{x}_r, \omega_i)} \\ \mathbf{G}_{yx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_x(\boldsymbol{x}_r, \omega_i)} + \mathbf{G}_{yy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_y(\boldsymbol{x}_r, \omega_i)} \end{bmatrix}$$

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- ullet Source, receivers and 1 defect small area around  ${m x}_d$  with different wave velocity
- Each time the original pulse passes from the defect it splits into a transmitted and a reflected component
- Assumption : the incident field  $\mathbf{v}_{inc}$  is known (response in the healthy domain)
- scattered filed  $\mathbf{v}_{scat} = \mathbf{v}_{tot} \mathbf{v}_{inc}$  to minimize the influence of the source
- The defect acts as a multiple in time source
- $\mathbf{v}_{scat}$  is time reversed and retransmitted
- Not only one refocusing time but the strongest at  $t_{RF} = T t_1 t_0$

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• Data at the receiver - Born approximation

$$\hat{\mathbf{v}}_{scat}(\boldsymbol{x}_r,\omega) = k^2 \hat{f}(\omega) \rho \hat{\mathbf{G}}(\boldsymbol{x}_s, \boldsymbol{x}_d, \omega) \hat{\mathbf{G}}(\boldsymbol{x}_d, \boldsymbol{x}_r, \omega)$$

• It seems natural to define the imaging functional as

$$I(\boldsymbol{x}) = \sum_{i,r} \hat{G}^{h}(\boldsymbol{x}, \boldsymbol{x}_{s}, \omega_{i}) \hat{G}^{h}(\boldsymbol{x}, \boldsymbol{x}_{r}, \omega_{i}) \overline{\hat{v}_{scat}(\boldsymbol{x}_{r}, \omega_{i})} \Delta \omega_{i}$$

• Equivalently to source localization, we compute

$$\begin{bmatrix} \mathbf{I}_x(\boldsymbol{x}) \\ \mathbf{I}_y(\boldsymbol{x}) \end{bmatrix} = \frac{\Delta\omega}{2\pi} \sum_{i,r} \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x}, \boldsymbol{x}_s, \omega_i) & \mathbf{G}_{xy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \\ \mathbf{G}_{yx}(\boldsymbol{x}, \boldsymbol{x}_s, \omega_i) & \mathbf{G}_{yy}(\boldsymbol{x}, \boldsymbol{x}_s, \omega_i) \end{bmatrix} \times \\ \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_x(\boldsymbol{x}_r, \omega_i)} + \mathbf{G}_{xy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_y(\boldsymbol{x}_r, \omega_i)} \\ \mathbf{G}_{yx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_x(\boldsymbol{x}_r, \omega_i)} + \mathbf{G}_{yy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \overline{\mathbf{v}_y(\boldsymbol{x}_r, \omega_i)} \end{bmatrix}$$

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#### 1 Introduction

- 2 Source localization
- 3 Defect localization
- 4 Numerical examples

#### 5 Conclusions

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- **Geometry :** rectangular domain  $L_x = 10$ . and  $L_y = 10$ .
- Mesh (numerical solution) : 200  $\times$  200 grid with rectangular elements
- Mesh (Imaging) : 50 × 50 grid with rectangular elements
- **Material** : elastic with Lamé coefficients  $\lambda = 1$ . and  $\mu = 1$ .
- Velocities : pressure waves c<sub>p</sub> = 1.73 and shear waves c<sub>s</sub> = 1.
- **Excitation function :** Ricker pulse at a central frequency 2.



1 source and 1 receiver - increasing total time T



 $1 \ {\rm source} \ {\rm and} \ 1 \ {\rm receiver}$  - increasing total time T



T = 0.5 diagonals, SNR = p1/p2 = 0.6534

 $1 \ {\rm source} \ {\rm and} \ 1 \ {\rm receiver}$  - increasing total time T

T = 1 diagonals, SNR = p1/p2 = 0.9678

 $1 \ {\rm source} \ {\rm and} \ 1 \ {\rm receiver}$  - increasing total time T

T = 2 diagonals, SNR = p1/p2 = 1.1967



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 $1 \ {\rm source} \ {\rm and} \ 1 \ {\rm receiver}$  - increasing total time T

T = 3 diagonals, SNR = p1/p2 = 1.4648



1 source and 1 receiver - increasing total time  ${\sf T}$ 

T = 5 diagonals, SNR = p1/p2 = 1.6664



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1 source and 1 receiver - increasing total time  ${\sf T}$ 

T = 10 diagonals, SNR = p1/p2 = 1.7472

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1 source and 1 receiver - increasing total time T

T = 20 diagonals, SNR = p1/p2 = 1.789



1 source and 1 receiver - increasing total time T

T = 30 diagonals, SNR = p1/p2 = 2.1821



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 $1 \ {\rm source} \ {\rm and} \ 1 \ {\rm receiver}$  - increasing total time T

T = 40 diagonals, SNR = p1/p2 = 2.392



1 source and 1 receiver - increasing total time  ${\sf T}$ 

T = 50 diagonals, SNR = p1/p2 = 2.44



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1 source and 1 receiver - increasing total time T



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1 source and increasing number of receivers



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1 source and increasing number of receivers



Nr = 1, T = 10 diagonals, SNR = 2.3351

1 source and increasing number of receivers



Nr = 2, T = 10 diagonals, SNR = 2.5843

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1 source and increasing number of receivers



Nr = 3, T = 10 diagonals, SNR = 3.5819

1 source and increasing number of receivers



Nr = 4, T = 10 diagonals, SNR = 4.2135

1 source and increasing number of receivers



Nr = 5, T = 10 diagonals, SNR = 4.7374

1 source and increasing number of receivers



Nr = 6, T = 10 diagonals, SNR = 4.8888

1 source and increasing number of receivers



Nr = 7, T = 10 diagonals, SNR = 5.3904

1 source and increasing number of receivers



Nr = 8, T = 10 diagonals, SNR = 5.5286

1 source and increasing number of receivers



Nr = 9, T = 10 diagonals, SNR = 5.9734

1 source and increasing number of receivers



Nr = 10, T = 10 diagonals, SNR = 5.7849

1 source and increasing number of receivers



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- **Geometry :** rectangular domain  $L_x = 10$ . and  $L_y = 10$ .
- Mesh (numerical solution) : 200  $\times$  200 grid with rectangular elements
- Mesh (Imaging) : 200  $\times$  200 grid with rectangular elements
- Material : elastic with Lamé coefficients  $\lambda = 1$ . and  $\mu = 1$ .
- Velocities : pressure waves  $c_p = 1.73$  and shear waves  $c_s = 1$ .
- Excitation function : Ricker pulse with central frequency 4.
- Defect :

location : (3.5,4.5) size : 0.05  $\times$  0.05 material :  $\lambda$  = 0.5 and  $\mu$  = 1.

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1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



# total field



1 defect, 1 source and array of 13 receivers



# total field



1 defect, 1 source and array of 13 receivers



total field



1 defect, 1 source and array of 13 receivers



total field



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1 defect, 1 source and array of 13 receivers



total field



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1 defect, 1 source and array of 13 receivers



total field



Image: A matrix

1 defect, 1 source and array of 13 receivers







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1 defect, 1 source and array of 13 receivers







1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and array of 13 receivers



1 defect, 1 source and increasing number of receivers



1 defect, 1 source and increasing number of receivers



1 defect, 1 source and increasing number of receivers



1 defect, 1 source and increasing number of receivers



1 defect, 1 source and increasing number of receivers



1 defect, 1 source and increasing number of receivers


1 defect, 1 source and increasing number of receivers



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1 defect, 1 source and increasing number of receivers



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1 defect, 1 source and increasing number of receivers



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1 defect, 1 source and increasing number of receivers



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- 2 Source localization
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- 4 Numerical examples
- 5 Conclusions

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### Summary and Conclusions

- Application of TR based imaging techniques <sup>6</sup> for source and scatterer localization in elastic bounded domains
- Very efficient compared to TR, the Green's functions are compute only once
- Difficulties in the elastic medium due to the two types of waves (pressure and shear) and their conversions
- Source localization :

sensor configuration : distributed or array steady increase and convergence of SNR for increasing total time T approximately linear increase of SNR for increasing number of sensors Boundaries : positive influence

• Defect localization :

sensor configuration : array total time T is very important, should be carefully chosen approximately linear increase of SNR for increasing number of sensors Boundaries : negative influence

<sup>6.</sup> L Borcea, G Papanicolaou, C Tsogka and J Berryman, (2002) Imaging and time reversal in random media

- Extensive investigation of the distributed sensor configuration
- Propose optimal total experiment time
- Investigation of the methodology using passive noisy recordings as input data

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- Account for dissipation (damping) and dispersion
- Application to structures with complex geometry

# Thank you!

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Image: A mathematical states of the state

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Image: A mathematical states of the state



Image: A mathematical states of the state



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Image: A matrix and a matrix



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Image: A mathematical states of the state

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HSTAM, May 2016 27 / 27

Image: A mathematical states of the state

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HSTAM, May 2016

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Image: A mathematical states of the state



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