# Damage detection and imaging in solids based on recorded elastodynamic response

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# Detection and Localization of Damage

- Usually based on response recordings at a number of sensors to monitor structural integrity [1]
- Detection : comparison of recordings to a reference (undamaged) state
- Localization : Inverse Problem usually ill-posed
- Solution : *Time-Reversal* (TR) computational tool introduced by Fink et. al. [2]
- Achieves refocusing of the wave on the source
- Sending back the recorded signals but reversed in time
- Two step approach
  - Forward step
  - Backward step

1. GE Stavroulakis, (2000) Inverse and crack identification problems in engineering mechanics
2. Fink et. al., (2000) Time-reversed acoustics

# Time Reversal and applications

- TR is a physical process
- It exploits the time reversibility (based on spatial reciprocity and time reversal invariance) of linear wave equations
- Robust and Simple technique for source localization
- Has been applied in Acoustics[3], Elastodynamics[4], Electromagnetism, Hydrodynamics etc.
- Finds several applications in medicine, telecommunications, underwater acoustics, seismology, engineering structures, etc.
- TR can be used for scatterer localization

example

3. L Borcea, G Papanicolaou, C Tsogka and J Berryman, (2002) Imaging and time reversal in random media
4. D Givoli, (2014) Time Reversal as a Computational Tool in Acoustics and Elastodynamics

# In the present work

- Imaging techniques that exploit the fundamental idea of TR
- Description of the numerical implementation for the elastic wave propagation
- Utilization of the Green's function of the Elastodynamic equation to apply imaging techniques
- 2D rectangular bounded domain with elastic behavior
- Investigation of the influence of the boundaries in imaging
- Investigation of the main factors that affect the quality of the image

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# Forward step

- Simulated Numerically using a mixed finite element formulation [5]
- The domain contains one source at $\boldsymbol{x}_s$ and $N_r$ receivers at $\boldsymbol{x}_r$, $r = 1, \ldots, N_r$
- Wave propagation model, Velocity - Stress (first order)

$$\rho \frac{\partial \mathbf{v}}{\partial t} - \operatorname{div} \sigma = \delta(\boldsymbol{x} - \boldsymbol{x}_s) f(t) \boldsymbol{e}_i$$

$$A : \frac{\partial \sigma}{\partial t} - \dot{\varepsilon} = 0 \qquad \dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \mathrm{v}_i}{\partial x_j} + \frac{\partial \mathrm{v}_j}{\partial x_i}\right)$$

- Homogeneous Neumann boundary conditions and zero initial conditions
- Excitation function $f(t)$ is a Ricker pulse centered at a known $t_0$
- The response is being recorded during total time $T$

5. E Bécache, P Joly and C Tsogka (2002) A new family of mixed finite elements for the linear elastodynamic problem.

# Time domain solution - TR Backward step

- Always performed numerically in SHM applications
- The recorded signal is time reversed and retransmitted at $\boldsymbol{x}_r$
- Sensors acting as sources introducing right hand side loading terms

$$\rho \frac{\partial \mathbf{v}^{TR}}{\partial t} - \operatorname{div} \sigma^{TR} = \sum_{r=1}^{N_r} \delta(\boldsymbol{x} - \boldsymbol{x}_r) \mathrm{v}(\boldsymbol{x}_r, T - t)$$

- Homogeneous Neumann boundary conditions and zero initial conditions
- Refocusing at time $t_{RF} = T - t_0$

# Frequency domain solution - Imaging

- Solution of the backward problem

$$\mathrm{F}(\boldsymbol{x}_r, t) = \mathrm{v}(\boldsymbol{x}_r, T - t) \Leftrightarrow \hat{\mathrm{F}}(\boldsymbol{x}_r, \omega) = \overline{\hat{\mathrm{v}}(\boldsymbol{x}_r, \omega)} e^{i\omega T}$$

# Frequency domain solution - Imaging

- Solution of the backward problem

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$$\mathrm{v}^{TR}(\boldsymbol{x}, t) = \mathrm{G}(\boldsymbol{x}, \boldsymbol{x}_r, t) \star_t \mathrm{F}(\boldsymbol{x}_r, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\mathrm{G}}(\boldsymbol{x}, \boldsymbol{x}_r, \omega) \hat{\mathrm{F}}(\boldsymbol{x}_r, \omega) \mathrm{d}\omega$$

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- Evaluation of $\mathrm{v}^{TR}(\boldsymbol{x}, t)$ at the refocusing time $T - t_0$

$$\mathrm{v}^{TR}(\boldsymbol{x}, t = T - t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\mathrm{G}}(\boldsymbol{x}, \boldsymbol{x}_r, \omega) \overline{\hat{\mathrm{v}}(\boldsymbol{x}_r, \omega)} e^{i\omega t_0} \mathrm{d}\omega$$

# Frequency domain solution - Imaging

- Solution of the backward problem

$$\mathrm{F}(\boldsymbol{x}_r,t)=\mathrm{v}(\boldsymbol{x}_r,T-t)\Leftrightarrow\hat{\mathrm{F}}(\boldsymbol{x}_r,\omega)=\overline{\hat{\mathrm{v}}(\boldsymbol{x}_r,\omega)}e^{i\omega T}$$

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- Imaging functional - numerical approximation

$$\boxed{\mathrm{I}(\boldsymbol{x})=\frac{1}{2\pi}\sum_{i,r}\hat{\mathrm{G}}^h(\boldsymbol{x},\boldsymbol{x}_r,\omega_i)\overline{\hat{\mathrm{v}}(\boldsymbol{x}_r,\omega_i)}\Delta\omega_i}$$

# Numerical implementation of Imaging

- More precisely

$$\mathrm{I}(\boldsymbol{x}) = \begin{bmatrix} \mathbf{I}_x(\boldsymbol{x}) \\ \mathbf{I}_y(\boldsymbol{x}) \end{bmatrix}, \quad \overline{\hat{\mathrm{v}}(\boldsymbol{x}_r, \omega_i)} = \begin{bmatrix} \overline{\mathrm{v}_x(\boldsymbol{x}_r, \omega_i)} \\ \overline{\mathrm{v}_y(\boldsymbol{x}_r, \omega_i)} \end{bmatrix},$$

$$\hat{\mathrm{G}}^h(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) = \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) & \mathbf{G}_{xy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \\ \mathbf{G}_{yx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) & \mathbf{G}_{yy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i) \end{bmatrix}$$

- The final image $\mathbf{I}$ can be a combination (e.g. SRSS) of $\mathbf{I}_x$ and $\mathbf{I}_y$, where

$$\begin{bmatrix} \mathbf{I}_x(\boldsymbol{x}) \\ \mathbf{I}_y(\boldsymbol{x}) \end{bmatrix} = \frac{\Delta\omega}{2\pi} \sum_{i,r} \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i)\overline{\mathrm{v}_x(\boldsymbol{x}_r, \omega_i)} + \mathbf{G}_{xy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i)\overline{\mathrm{v}_y(\boldsymbol{x}_r, \omega_i)} \\ \mathbf{G}_{yx}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i)\overline{\mathrm{v}_x(\boldsymbol{x}_r, \omega_i)} + \mathbf{G}_{yy}(\boldsymbol{x}, \boldsymbol{x}_r, \omega_i)\overline{\mathrm{v}_y(\boldsymbol{x}_r, \omega_i)} \end{bmatrix}$$

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# Time domain solution - TR

- Source, receivers and 1 defect - small area around $\boldsymbol{x}_d$ with different wave velocity
- Each time the original pulse passes from the defect it splits into a transmitted and a reflected component
- Assumption : the incident field $\mathbf{v}_{inc}$ is known (response in the healthy domain)
- scattered filed $\mathbf{v}_{scat} = \mathbf{v}_{tot} - \mathbf{v}_{inc}$ to minimize the influence of the source
- The defect acts as a multiple in time source
- $\mathbf{v}_{scat}$ is time reversed and retransmitted
- Not only one refocusing time but the strongest at $t_{RF} = T - t_1 - t_0$

# Frequency domain solution - Imaging

- Data at the receiver - Born approximation

$$\hat{\mathrm{v}}_{scat}(\boldsymbol{x}_r,\omega) = k^2 \hat{f}(\omega)\rho \hat{\mathrm{G}}(\boldsymbol{x}_s,\boldsymbol{x}_d,\omega)\hat{\mathrm{G}}(\boldsymbol{x}_d,\boldsymbol{x}_r,\omega)$$

- It seems natural to define the imaging functional as

$$\boxed{\mathrm{I}(\boldsymbol{x}) = \sum_{i,r} \hat{\mathrm{G}}^h(\boldsymbol{x},\boldsymbol{x}_s,\omega_i)\hat{\mathrm{G}}^h(\boldsymbol{x},\boldsymbol{x}_r,\omega_i)\overline{\hat{\mathrm{v}}_{scat}(\boldsymbol{x}_r,\omega_i)}\Delta\omega_i}$$

- Equivalently to source localization, we compute

$$\begin{bmatrix} \mathbf{I}_x(\boldsymbol{x}) \\ \mathbf{I}_y(\boldsymbol{x}) \end{bmatrix} = \frac{\Delta\omega}{2\pi}\sum_{i,r}\begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x},\boldsymbol{x}_s,\omega_i) & \mathbf{G}_{xy}(\boldsymbol{x},\boldsymbol{x}_r,\omega_i) \\ \mathbf{G}_{yx}(\boldsymbol{x},\boldsymbol{x}_s,\omega_i) & \mathbf{G}_{yy}(\boldsymbol{x},\boldsymbol{x}_s,\omega_i) \end{bmatrix} \times \begin{bmatrix} \mathbf{G}_{xx}(\boldsymbol{x},\boldsymbol{x}_r,\omega_i)\overline{\mathrm{v}_x(\boldsymbol{x}_r,\omega_i)} + \mathbf{G}_{xy}(\boldsymbol{x},\boldsymbol{x}_r,\omega_i)\overline{\mathrm{v}_y(\boldsymbol{x}_r,\omega_i)} \\ \mathbf{G}_{yx}(\boldsymbol{x},\boldsymbol{x}_r,\omega_i)\overline{\mathrm{v}_x(\boldsymbol{x}_r,\omega_i)} + \mathbf{G}_{yy}(\boldsymbol{x},\boldsymbol{x}_r,\omega_i)\overline{\mathrm{v}_y(\boldsymbol{x}_r,\omega_i)} \end{bmatrix}$$

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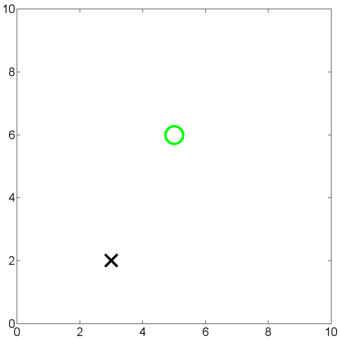
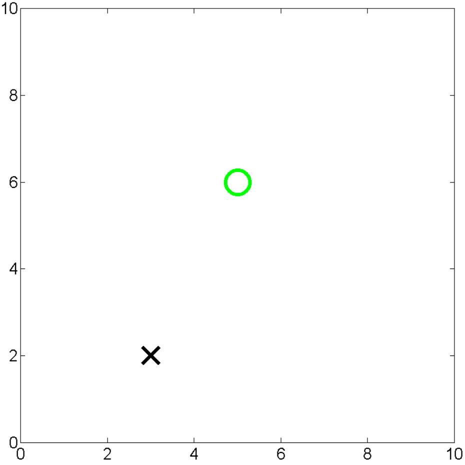
# Numerical example - source localization

- **Geometry :** rectangular domain
  $L_x = 10.$ and $L_y = 10.$
- **Mesh (numerical solution) :** $200 \times 200$ grid with rectangular elements
- **Mesh (Imaging) :** $50 \times 50$ grid with rectangular elements
- **Material :** elastic with Lamé coefficients
  $\lambda = 1.$ and $\mu = 1.$
- **Velocities :** pressure waves $c_p = 1.73$ and shear waves $c_s = 1.$
- **Excitation function :** Ricker pulse at a central frequency 2.



# Numerical example - source localization

1 source and 1 receiver - increasing total time T



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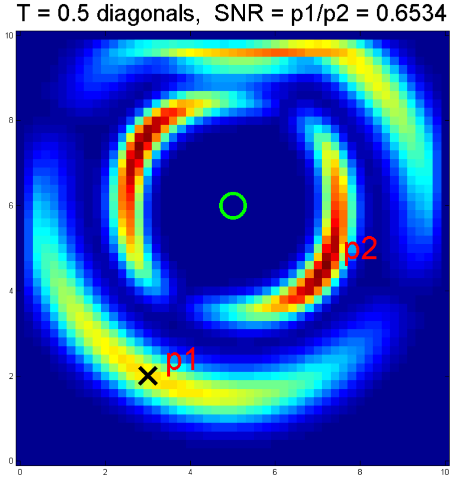

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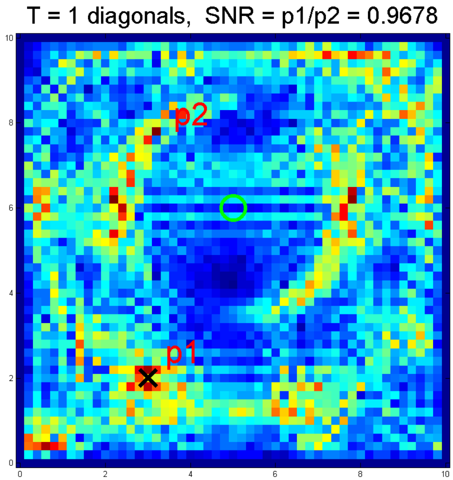

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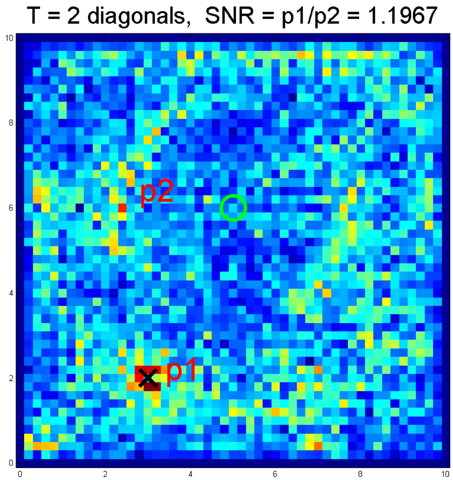

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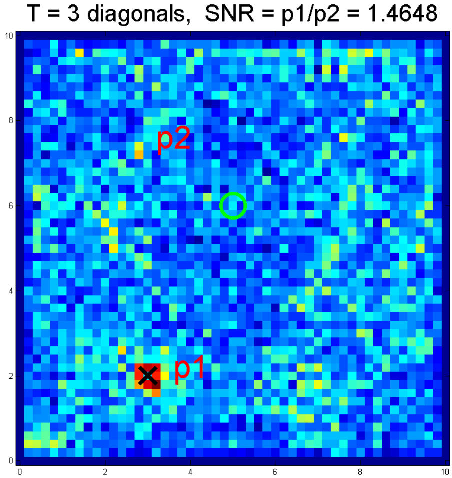

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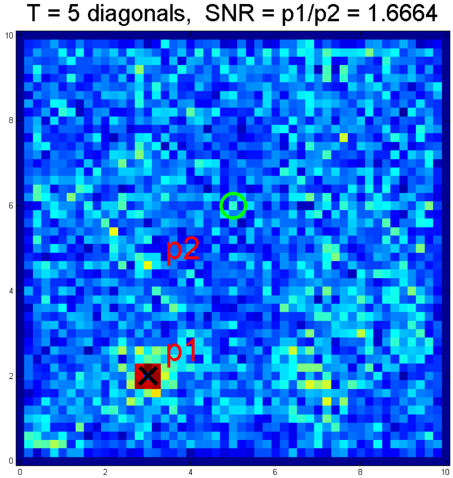

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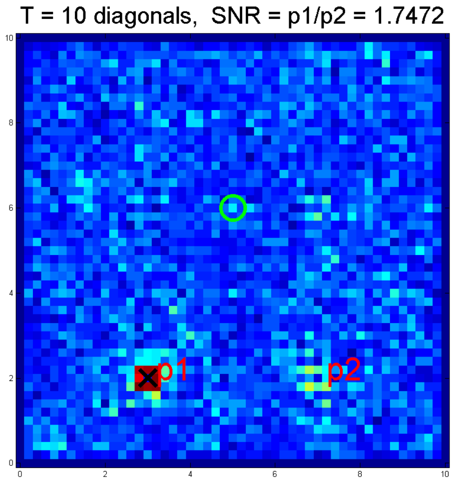

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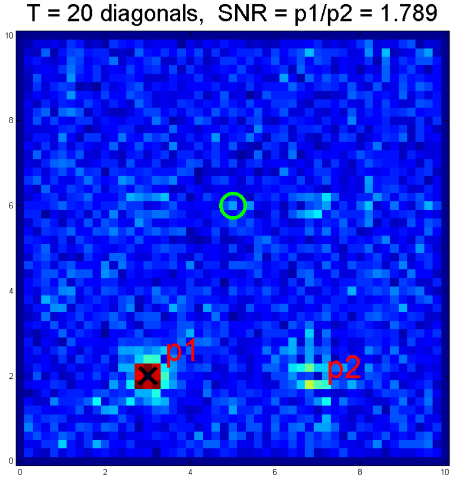

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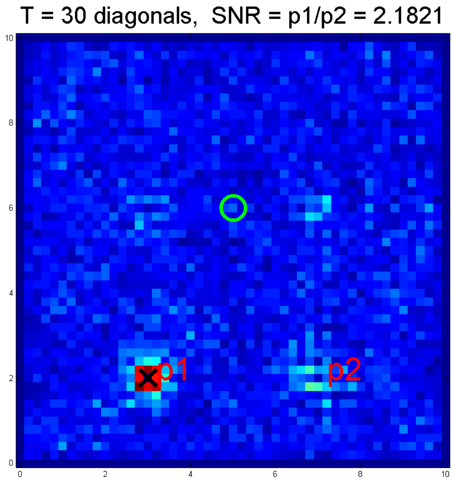

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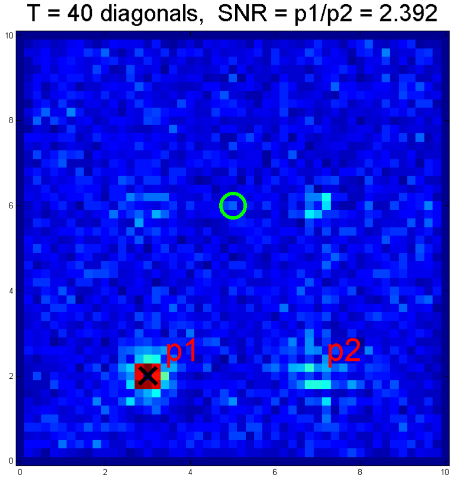

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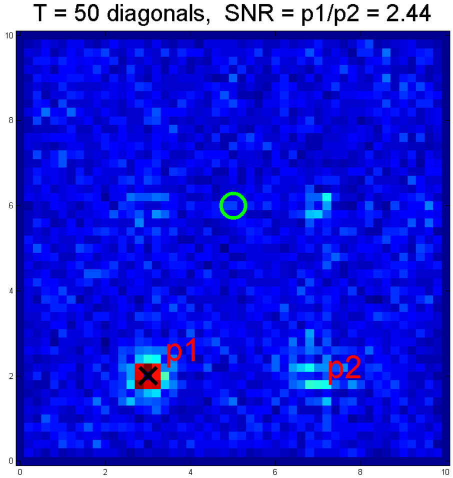

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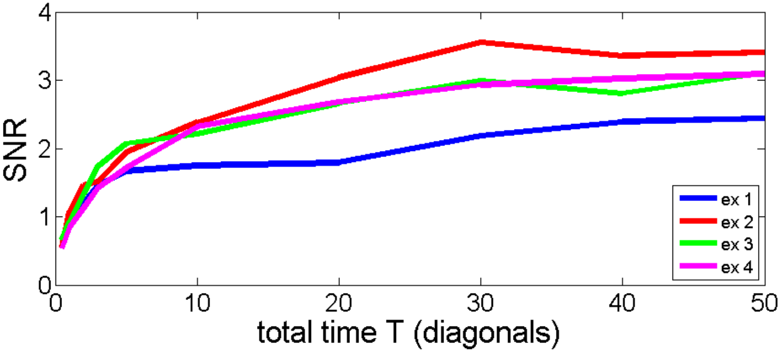

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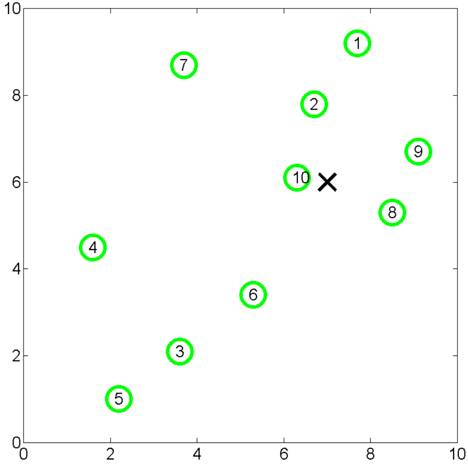
1 source and increasing number of receivers



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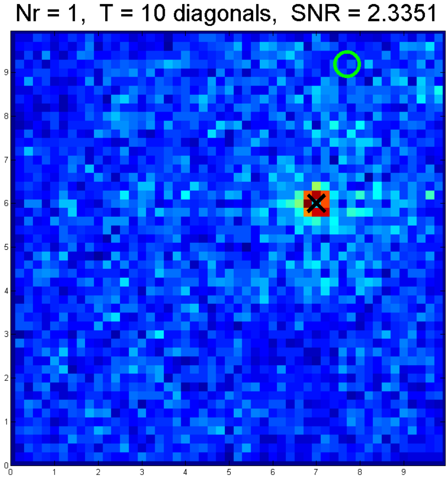

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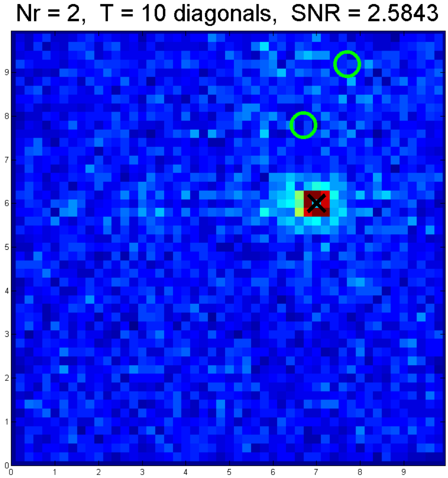

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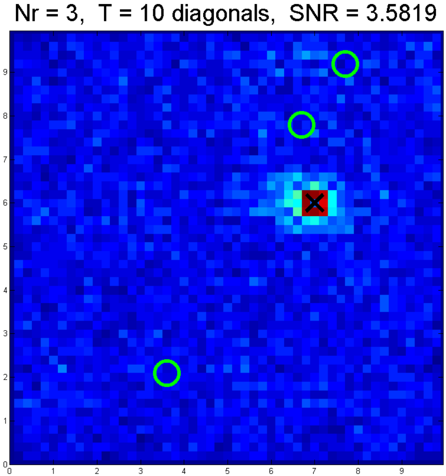

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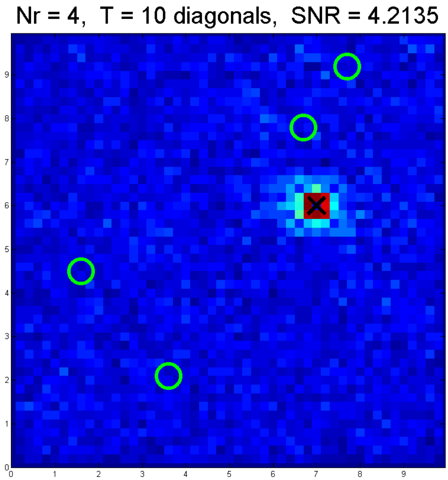

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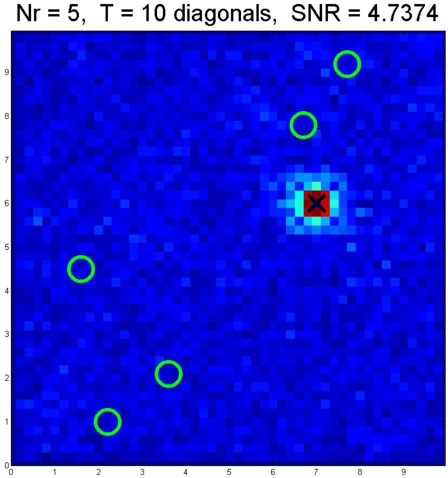

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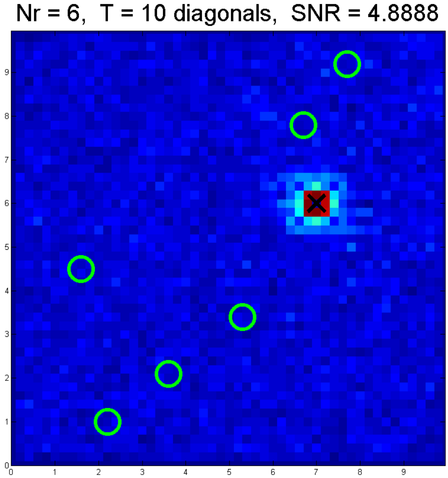

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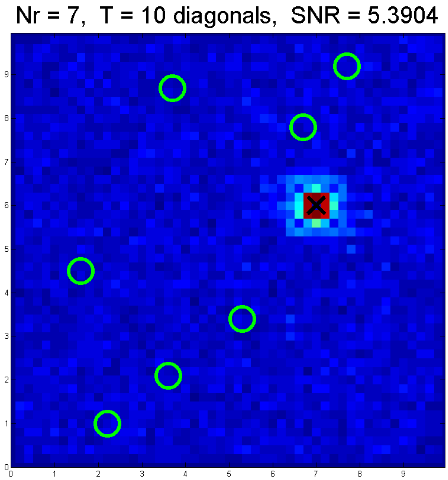

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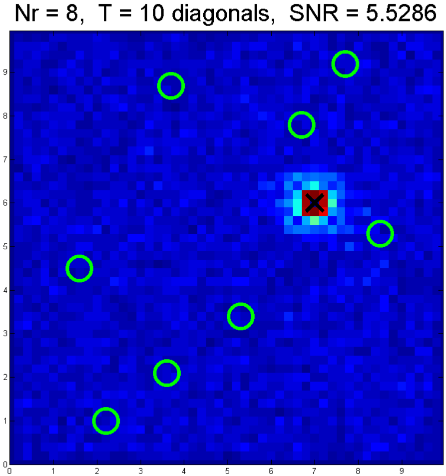

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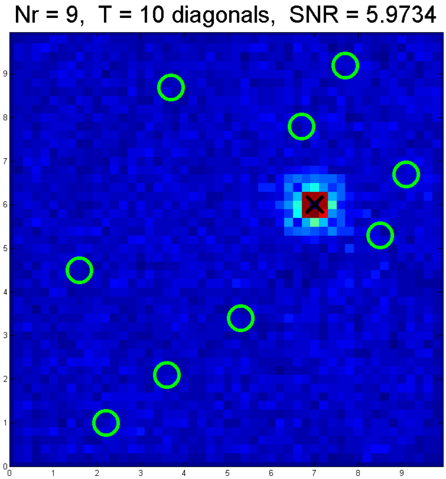
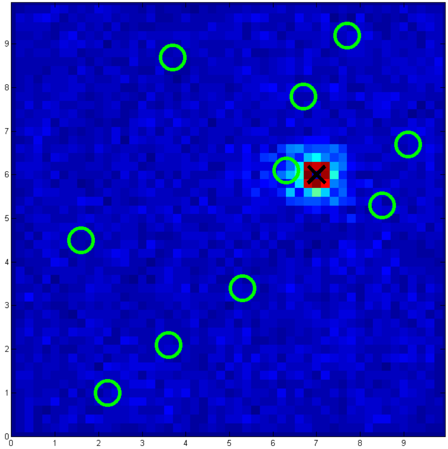


1 source and increasing number of receivers

Nr = 10, T = 10 diagonals, SNR = 5.7849



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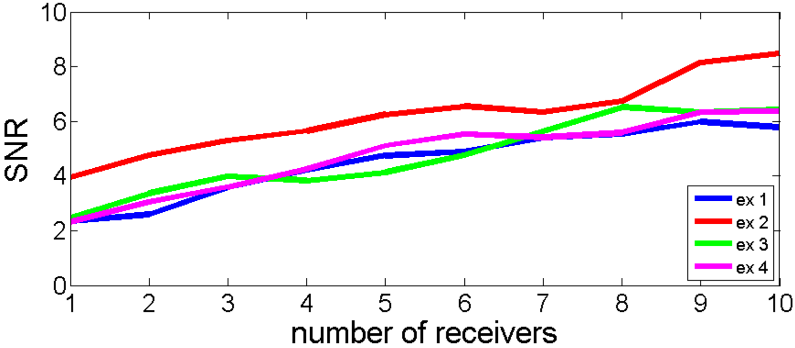


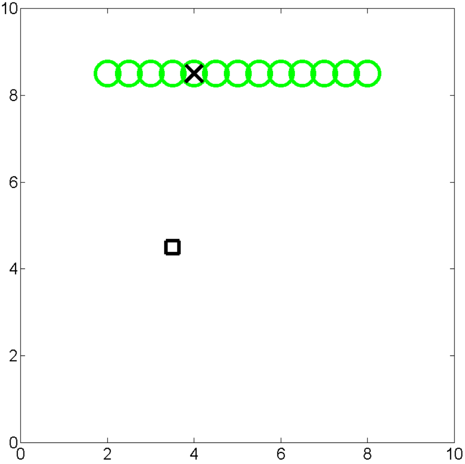
# Numerical example - defect localization

- **Geometry :** rectangular domain $L_x = 10.$ and $L_y = 10.$
- **Mesh (numerical solution) :** $200 \times 200$ grid with rectangular elements
- **Mesh (Imaging) :** $200 \times 200$ grid with rectangular elements
- **Material :** elastic with Lamé coefficients $\lambda = 1.$ and $\mu = 1.$
- **Velocities :** pressure waves $c_p = 1.73$ and shear waves $c_s = 1.$
- **Excitation function :** Ricker pulse with central frequency 4.
- **Defect :**

  location : (3.5,4.5)
  size : $0.05 \times 0.05$
  material : $\lambda = 0.5$ and $\mu = 1.$

# Numerical example - defect localization

1 defect, 1 source and array of 13 receivers



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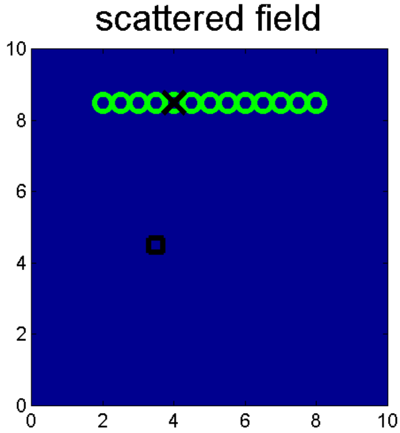


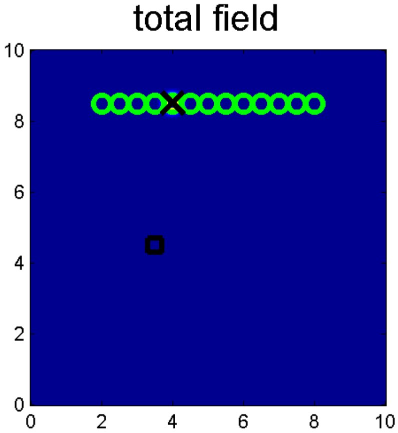

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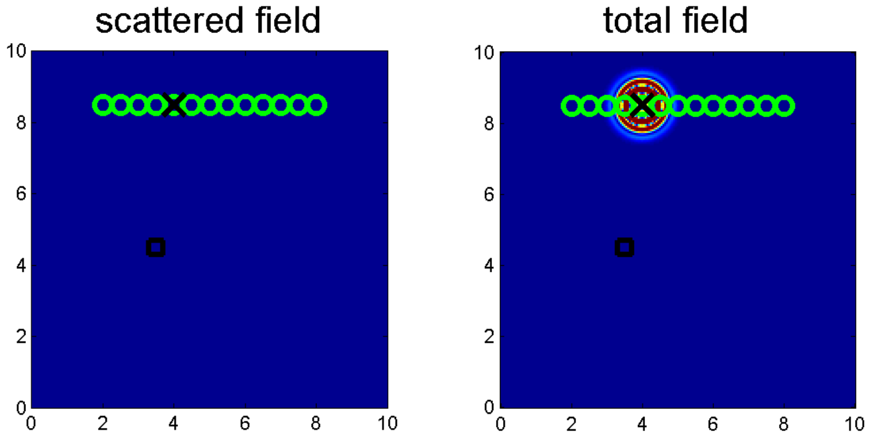

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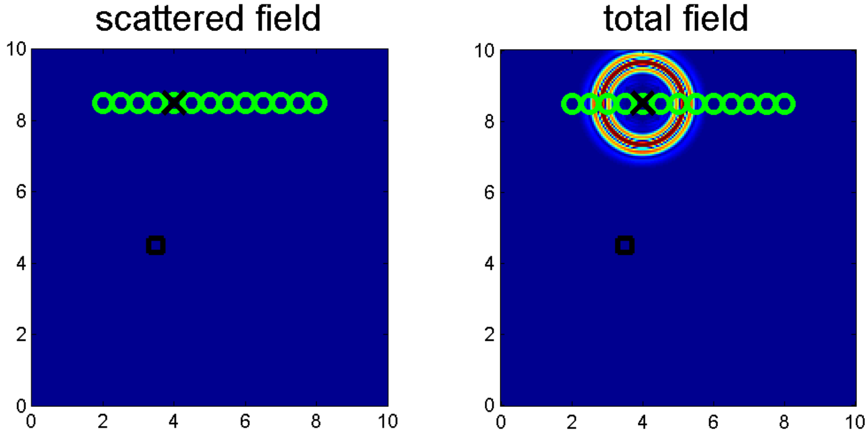

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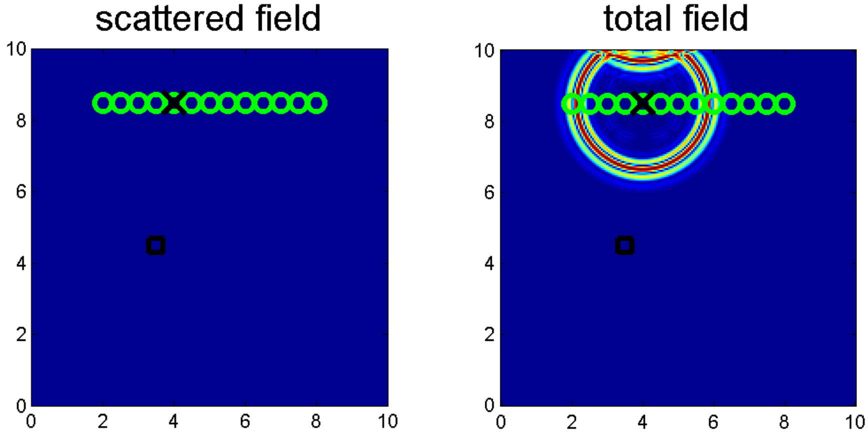

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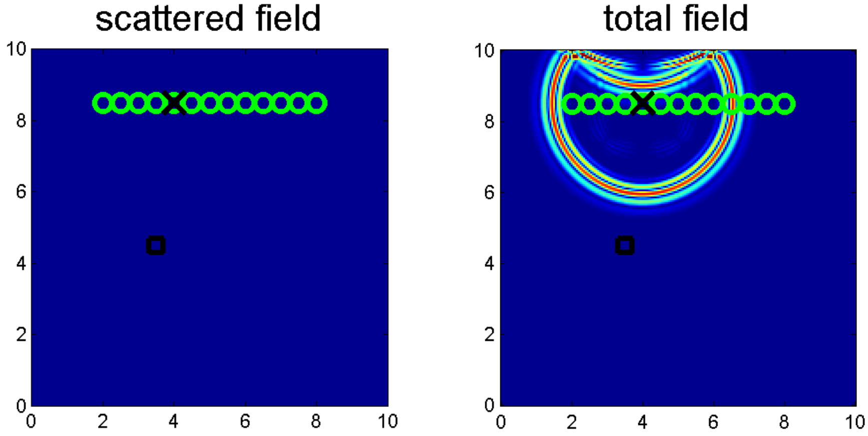

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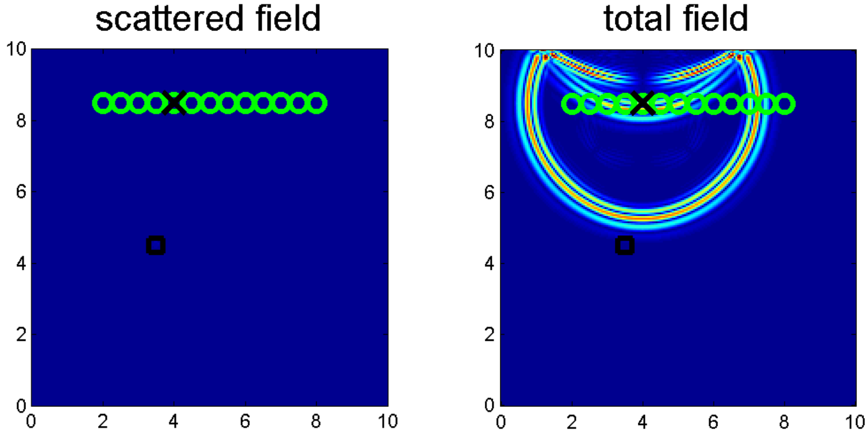

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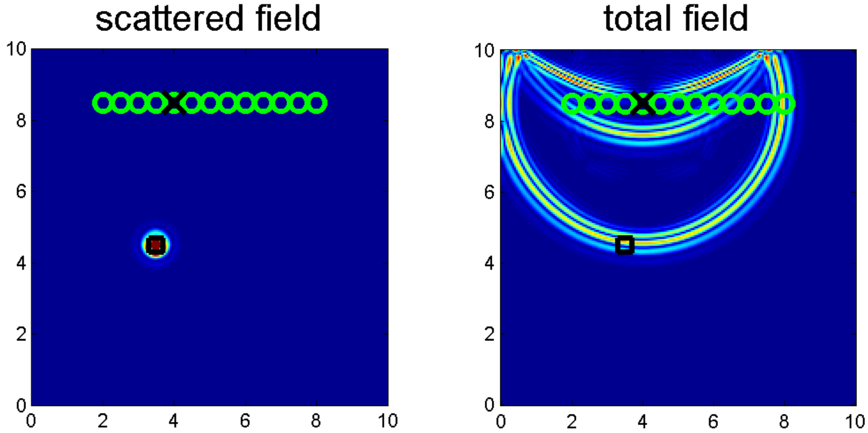
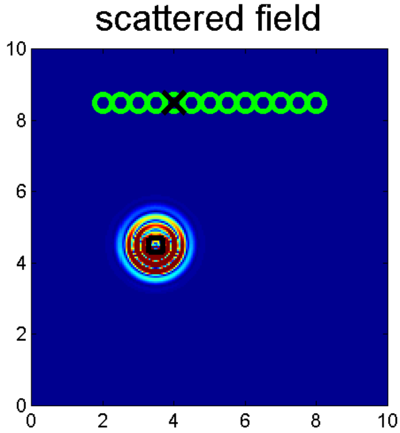
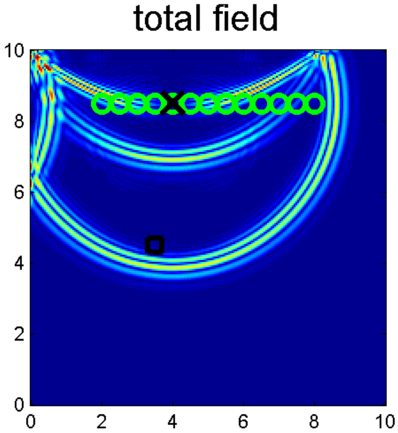


# Numerical example - defect localization

1 defect, 1 source and array of 13 receivers



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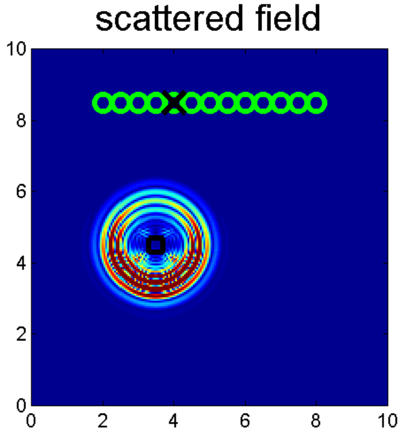
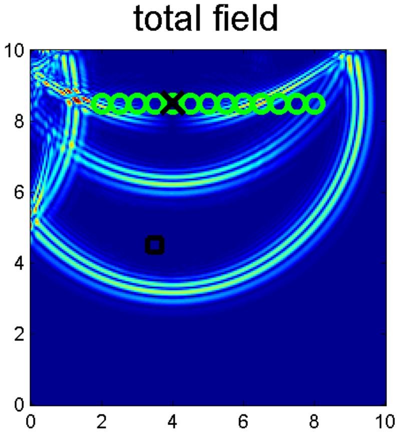
1 defect, 1 source and array of 13 receivers



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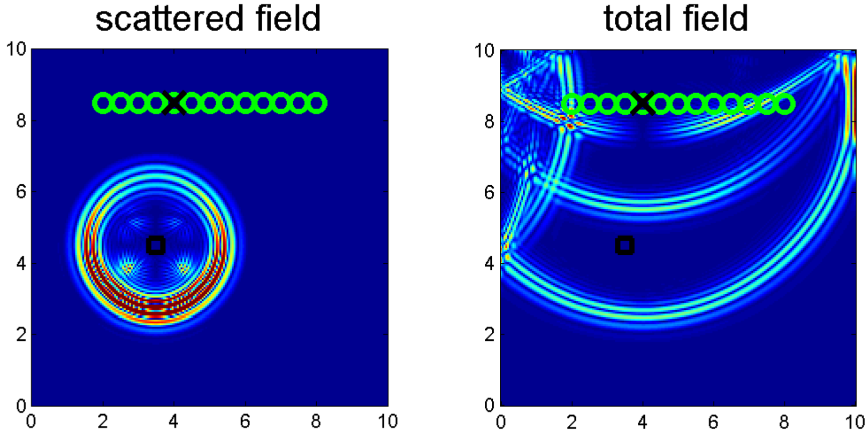

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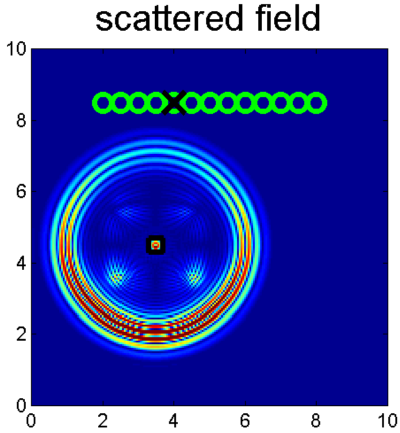


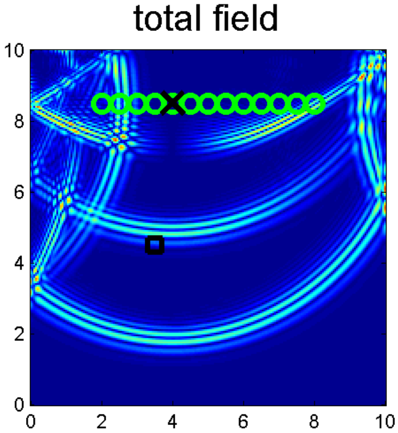

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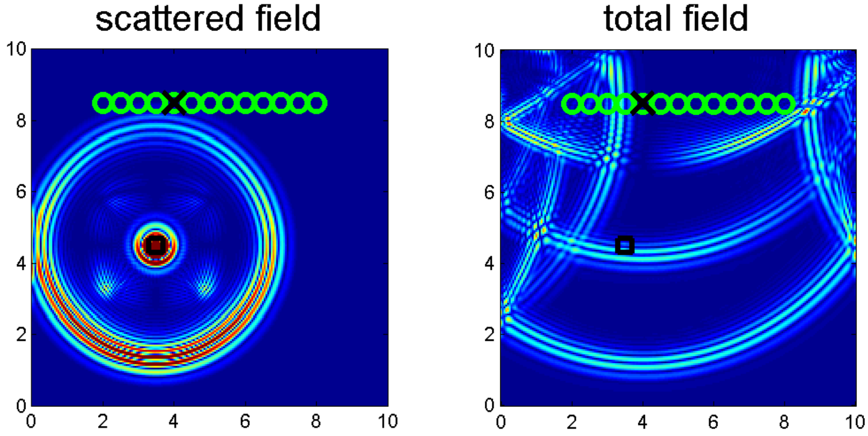

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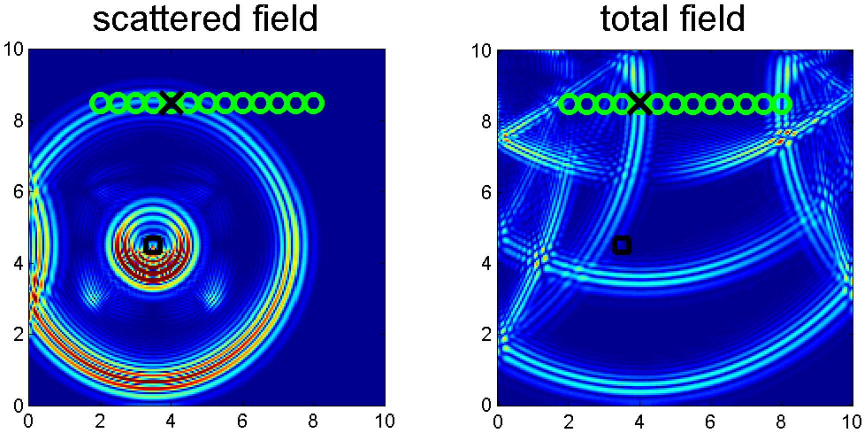

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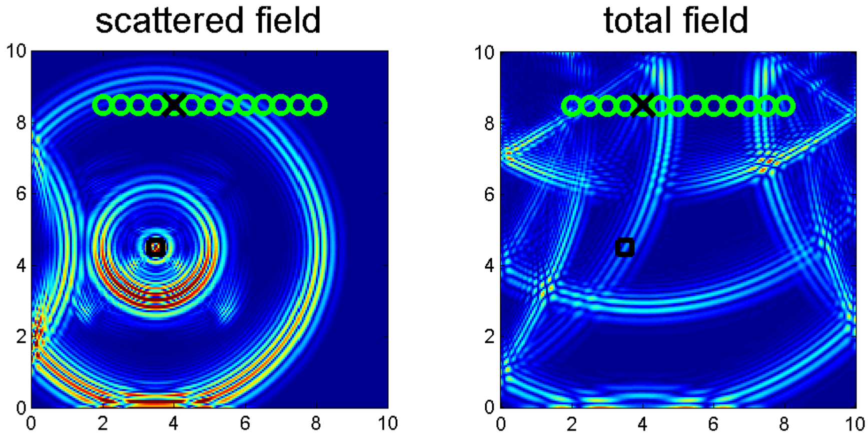

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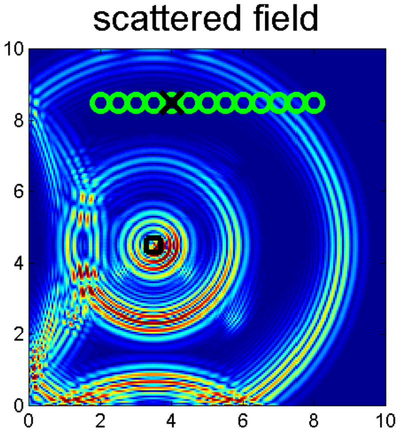


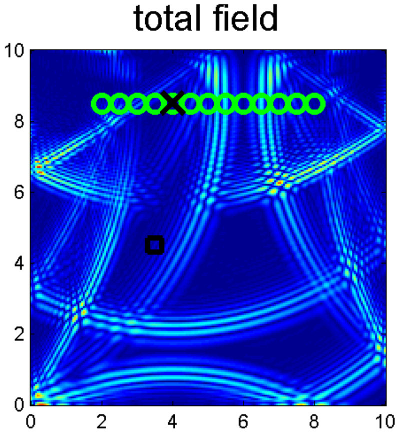

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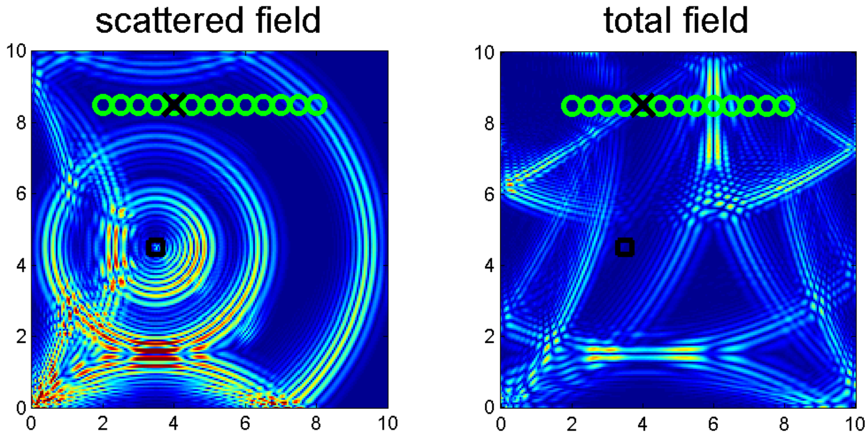

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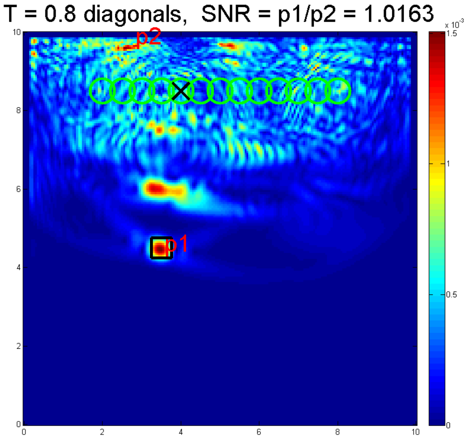

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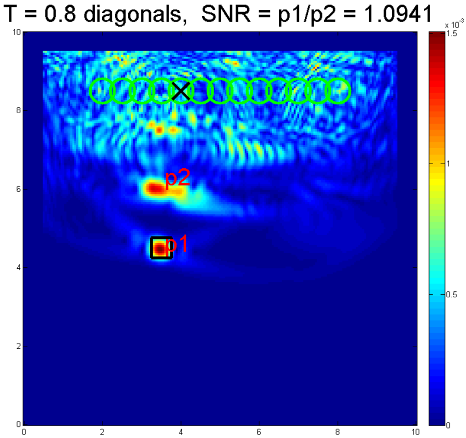

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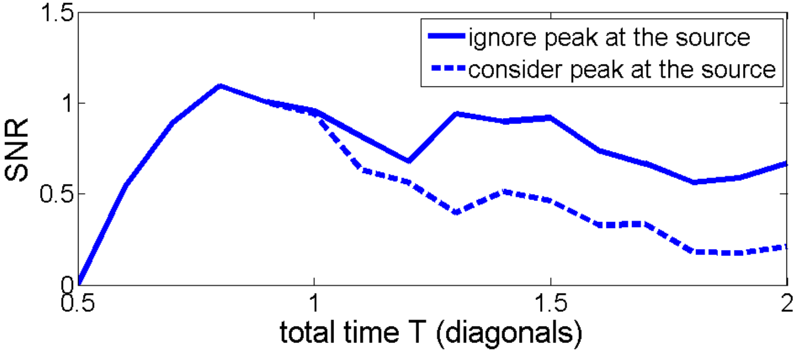
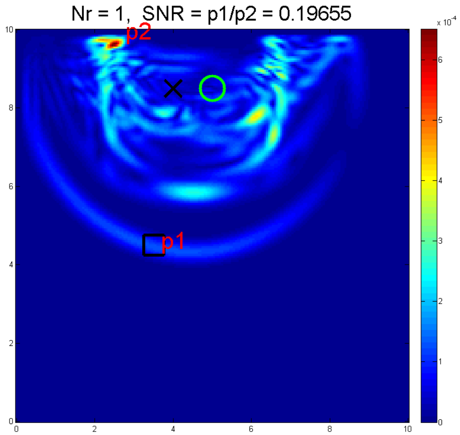
1 defect, 1 source and array of 13 receivers



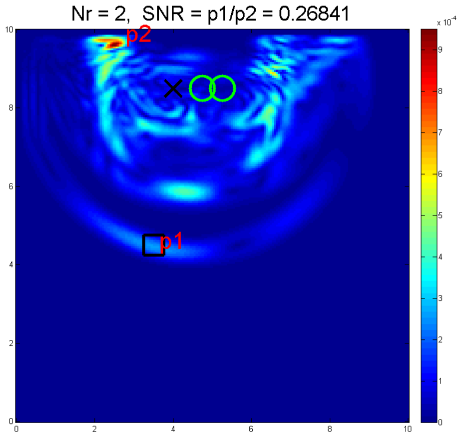
# Numerical example - defect localization

1 defect, 1 source and increasing number of receivers



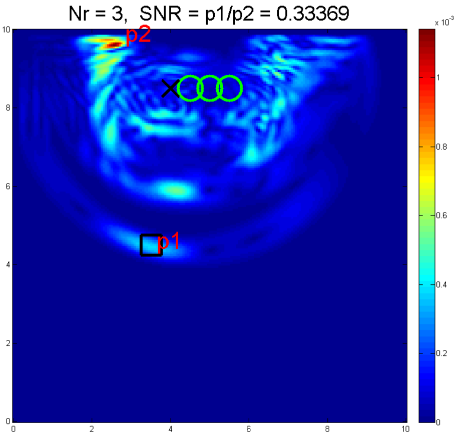
# Numerical example - defect localization

1 defect, 1 source and increasing number of receivers



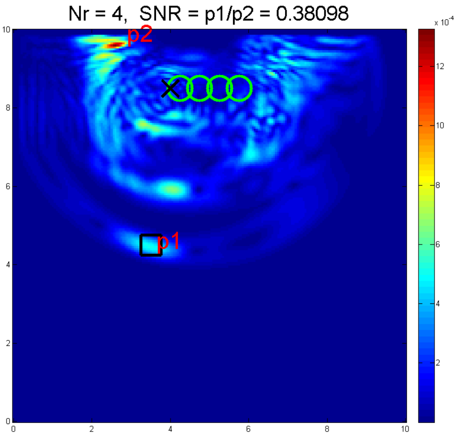
# Numerical example - defect localization

1 defect, 1 source and increasing number of receivers



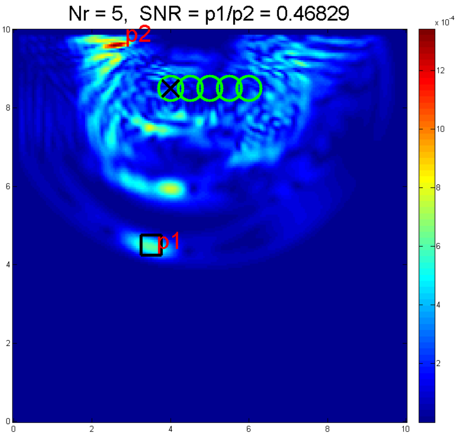
# Numerical example - defect localization

1 defect, 1 source and increasing number of receivers



# Numerical example - defect localization

1 defect, 1 source and increasing number of receivers



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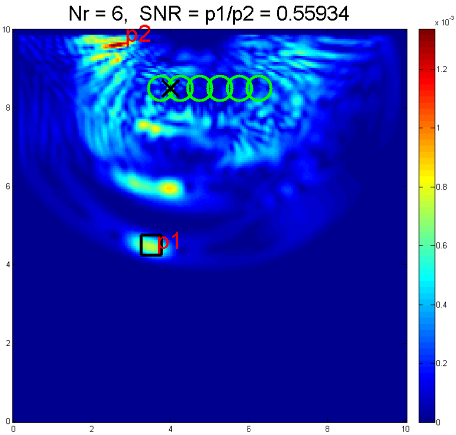

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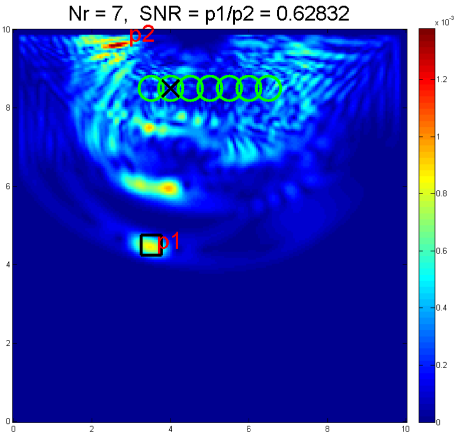

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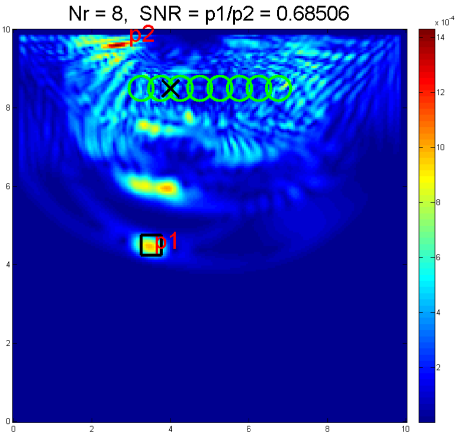

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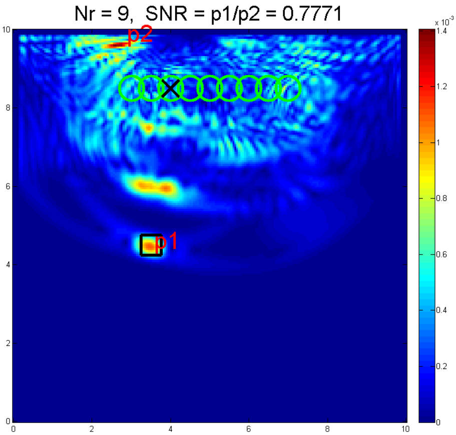

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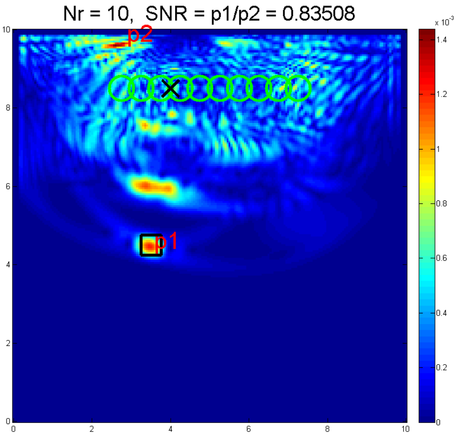

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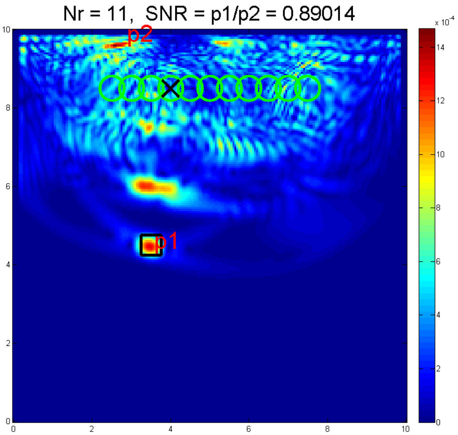

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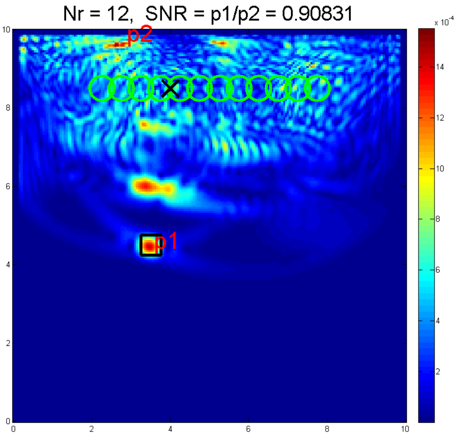

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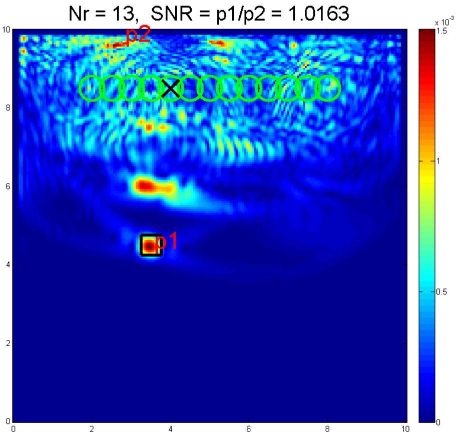

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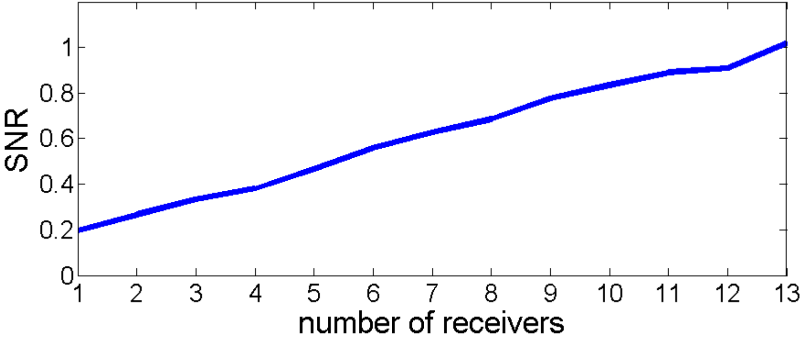


# Table of contents

# Summary and Conclusions

- Application of TR based imaging techniques [6] for source and scatterer localization in elastic bounded domains
- Very efficient compared to TR, the Green's functions are compute only once
- Difficulties in the elastic medium due to the two types of waves (pressure and shear) and their conversions
- Source localization :

  sensor configuration : distributed or array
  steady increase and convergence of SNR for increasing total time T
  approximately linear increase of SNR for increasing number of sensors
  Boundaries : positive influence
- Defect localization :

  sensor configuration : array
  total time T is very important, should be carefully chosen
  approximately linear increase of SNR for increasing number of sensors
  Boundaries : negative influence

---

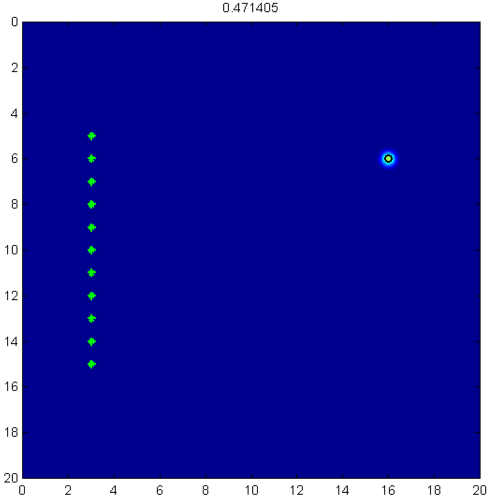
6. L Borcea, G Papanicolaou, C Tsogka and J Berryman, (2002) Imaging and time reversal in random media

# Future work

- Extensive investigation of the distributed sensor configuration
- Propose optimal total experiment time
- Investigation of the methodology using passive noisy recordings as input data
- Account for dissipation (damping) and dispersion
- Application to structures with complex geometry

Thank you !

# Source localization - Forward step



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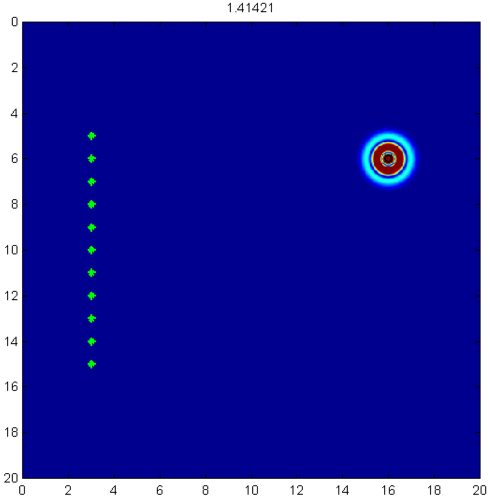
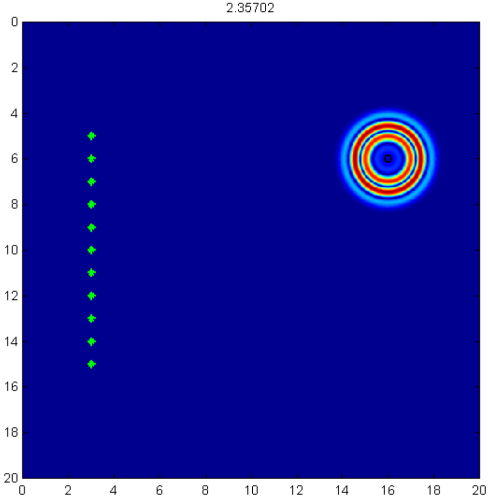



main

# Source localization - Forward step

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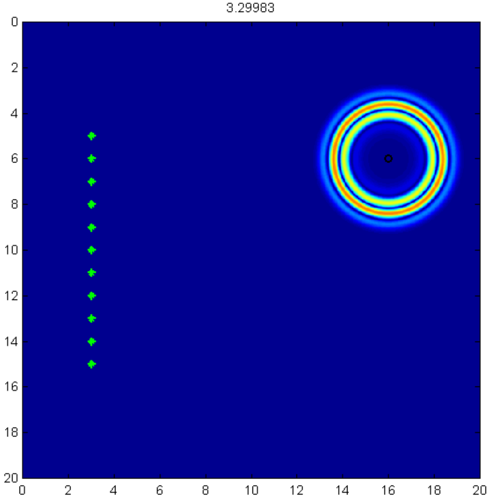


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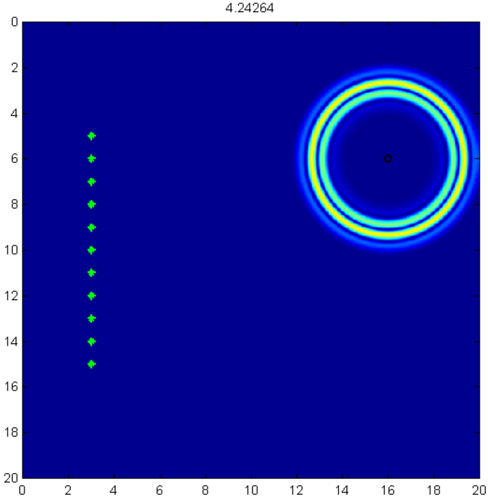


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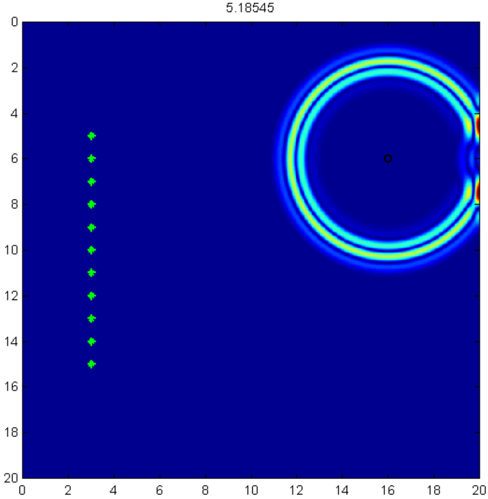


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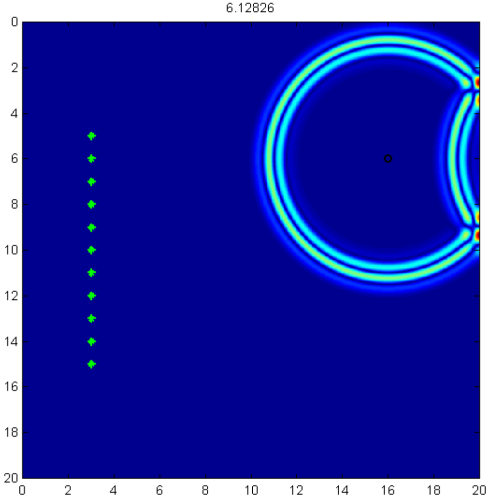


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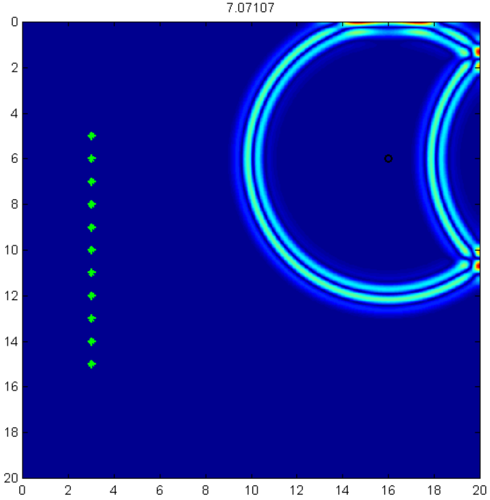


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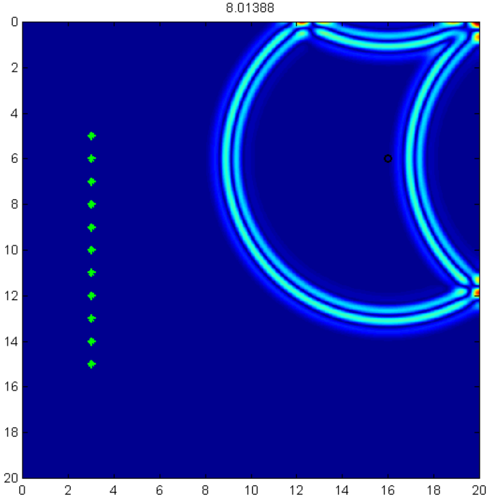


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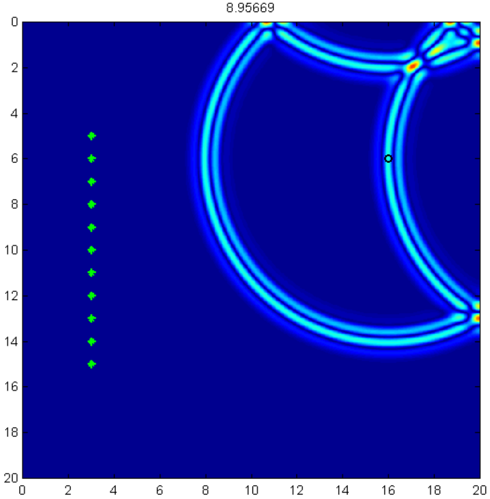

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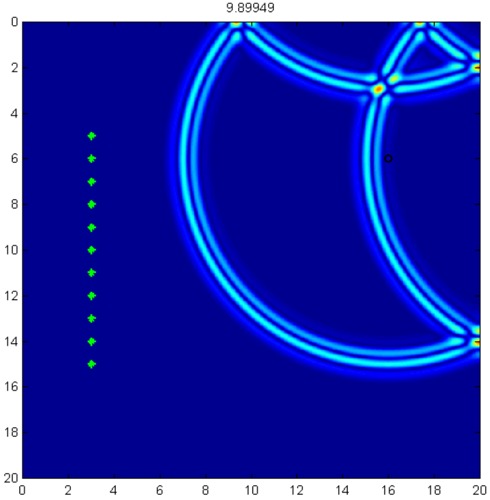

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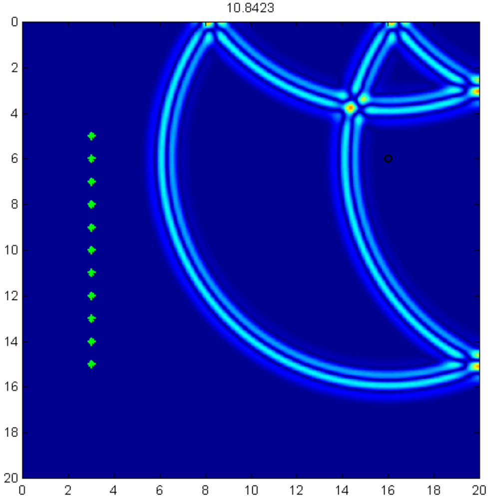

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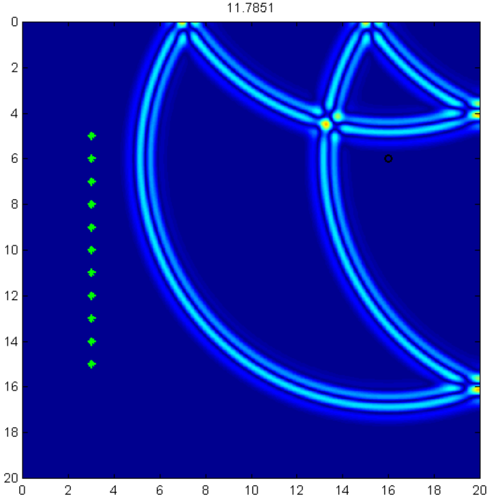


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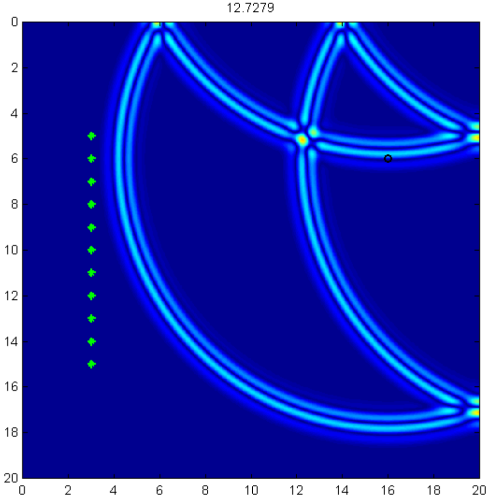


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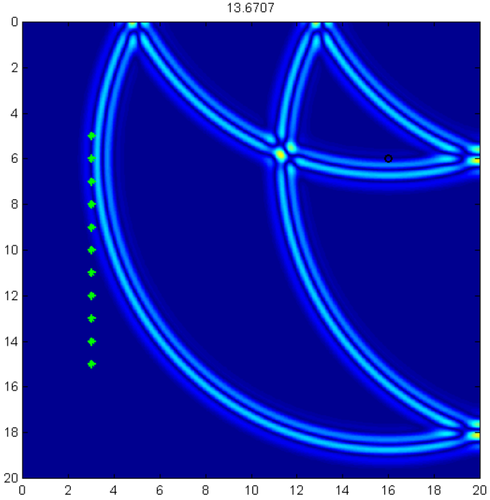

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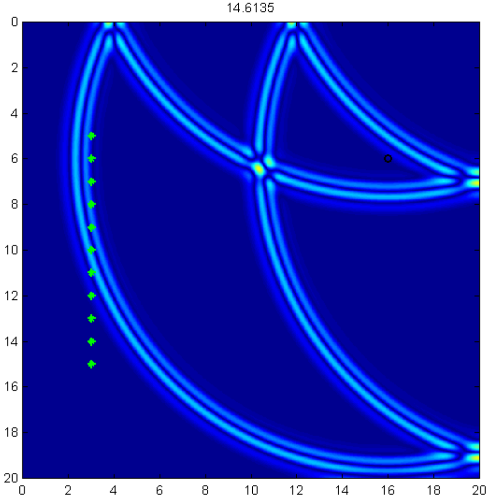


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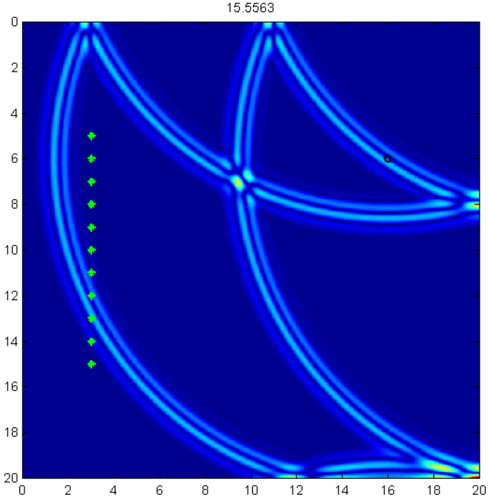


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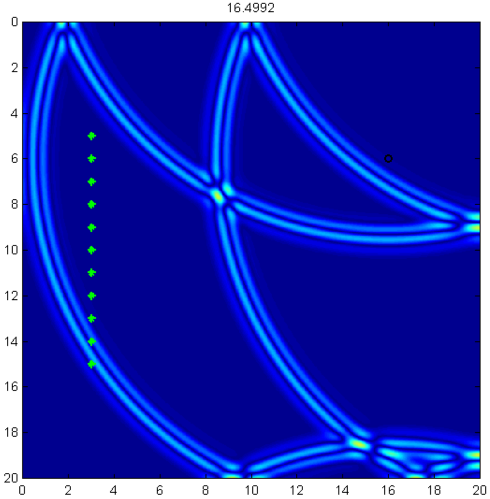


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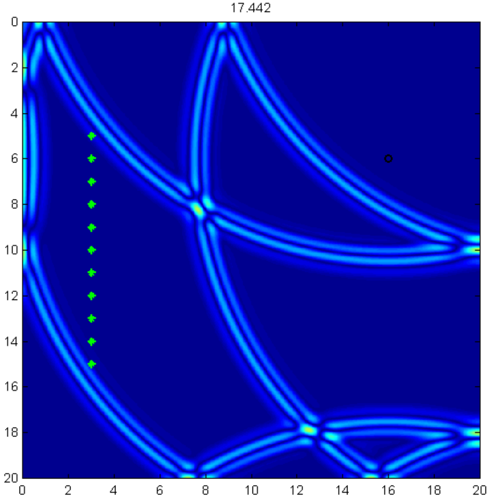

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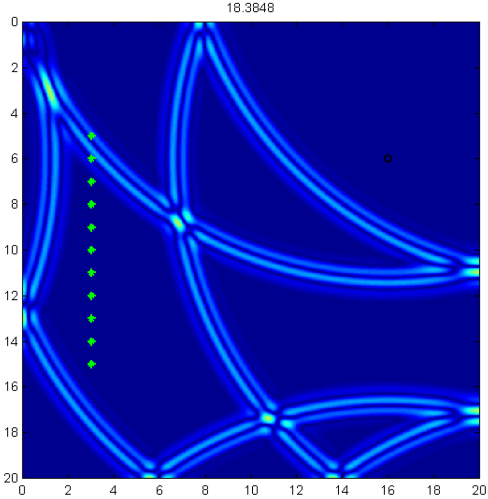


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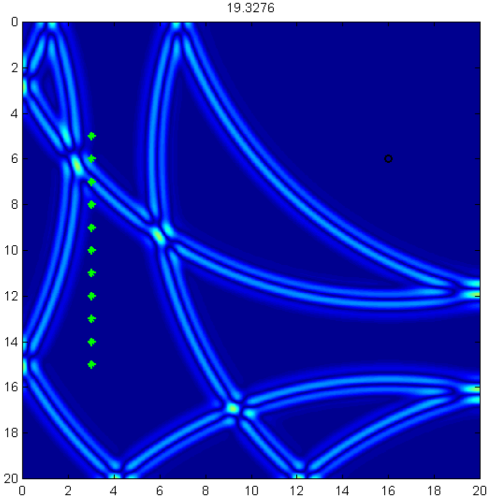


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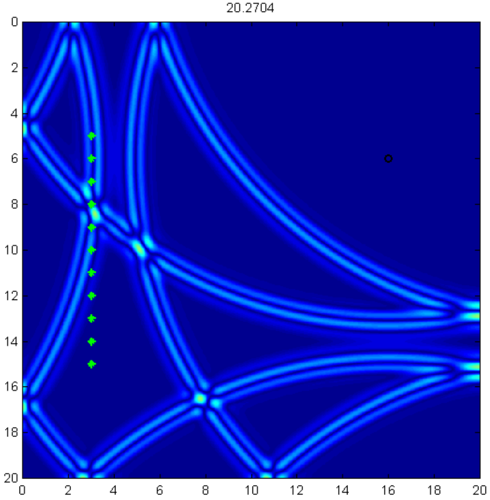


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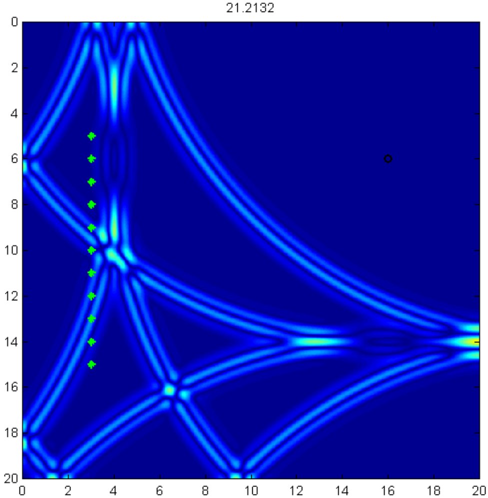


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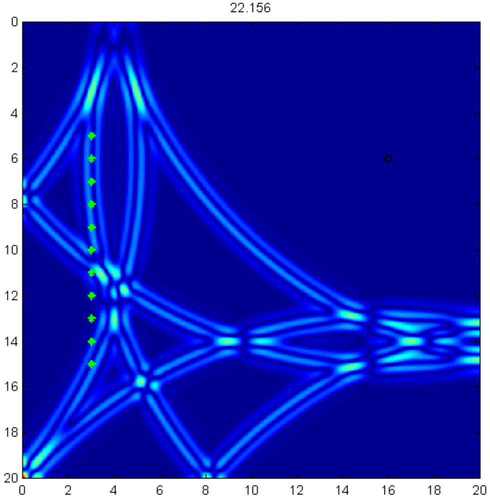

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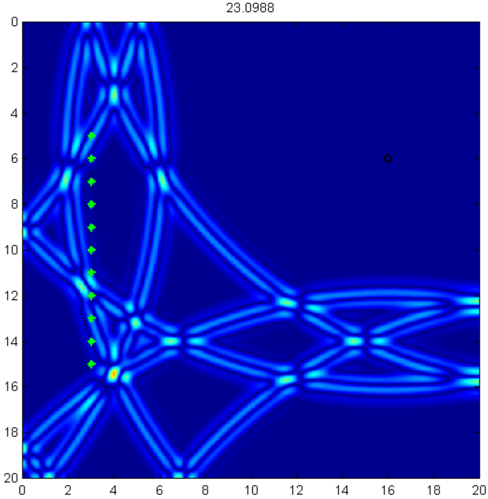


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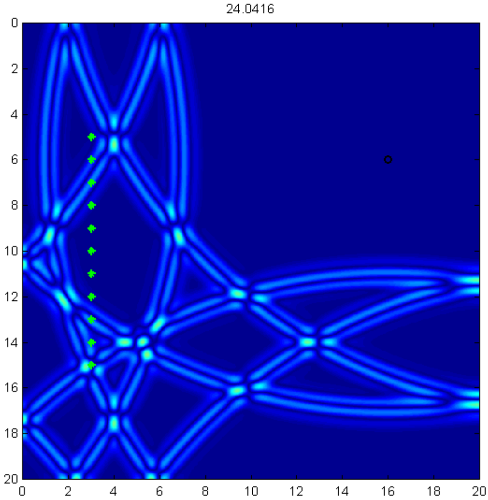


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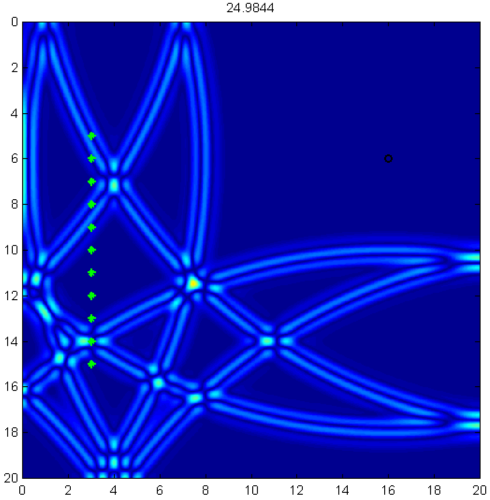


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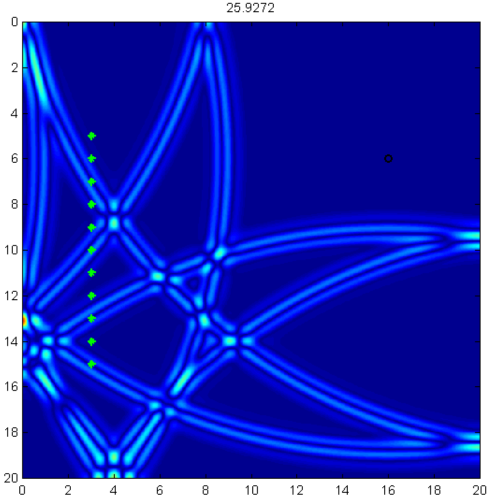


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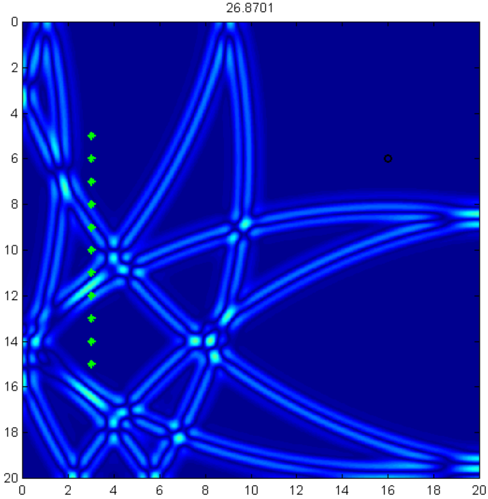

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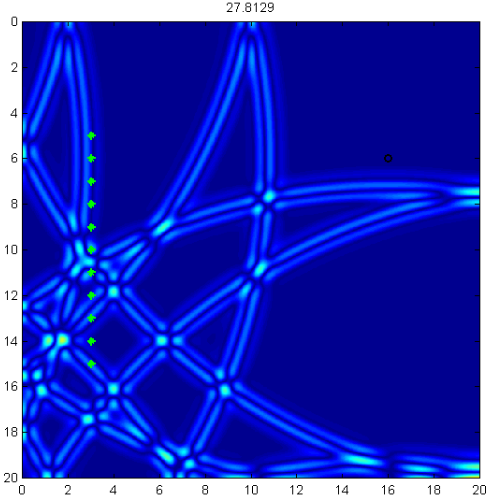


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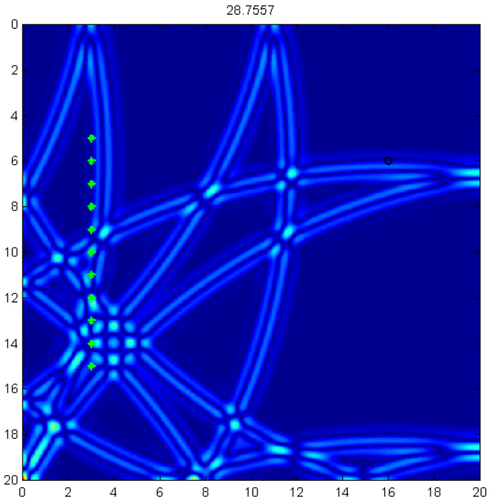


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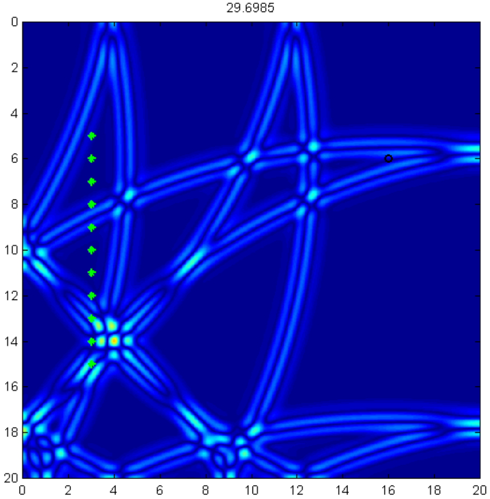


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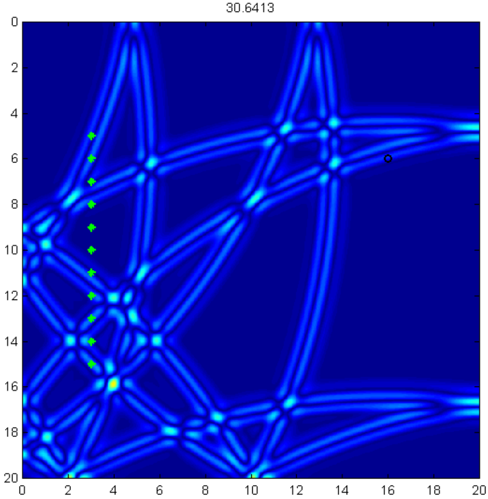

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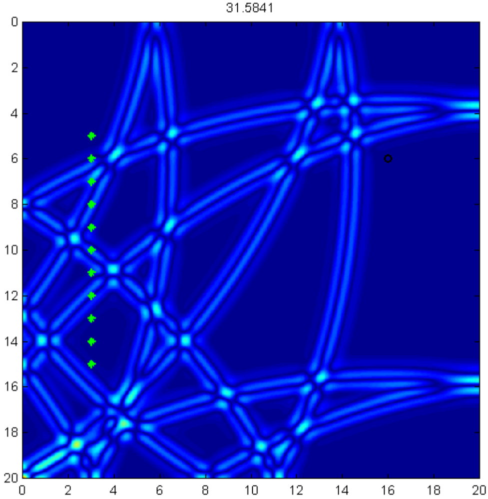


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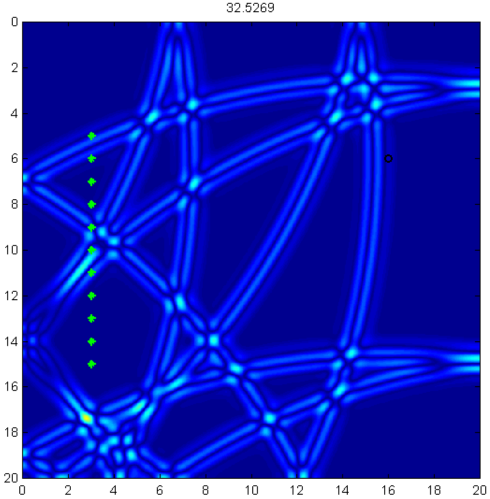


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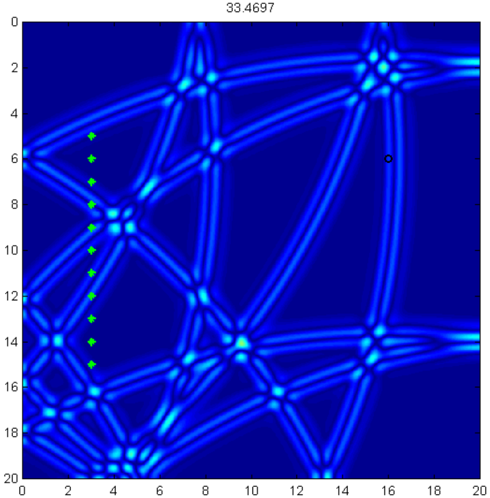


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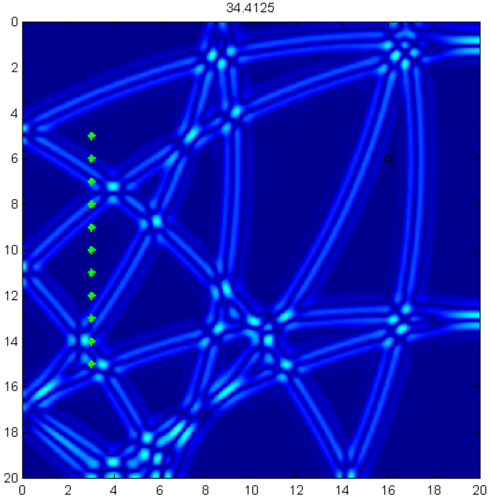
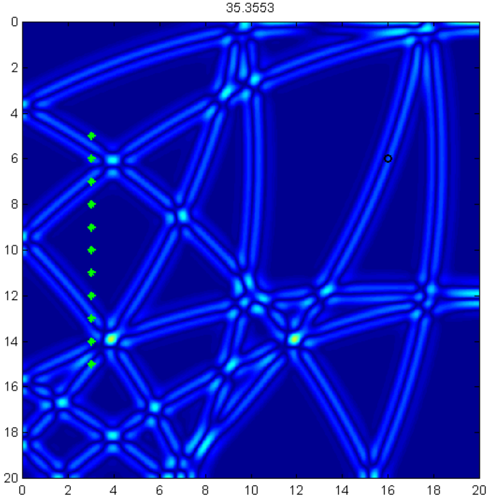


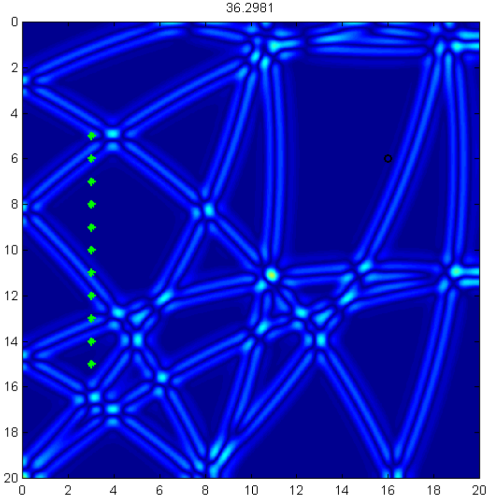

main

# Source localization - Forward step



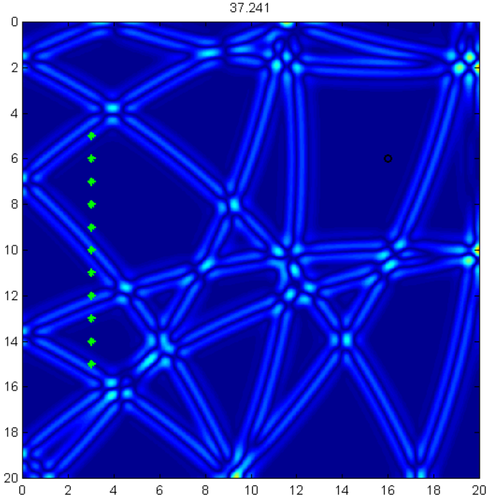

main

# Source localization - Forward step

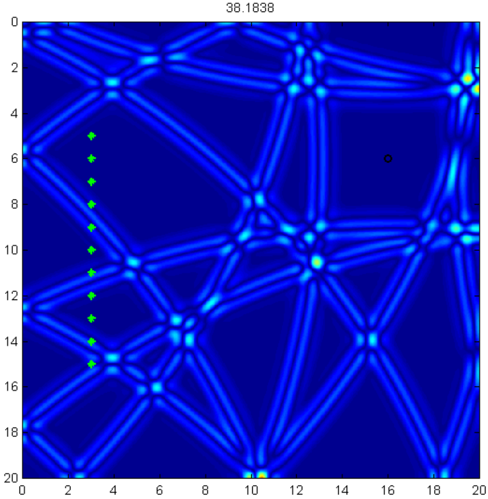



main

# Source localization - Forward step

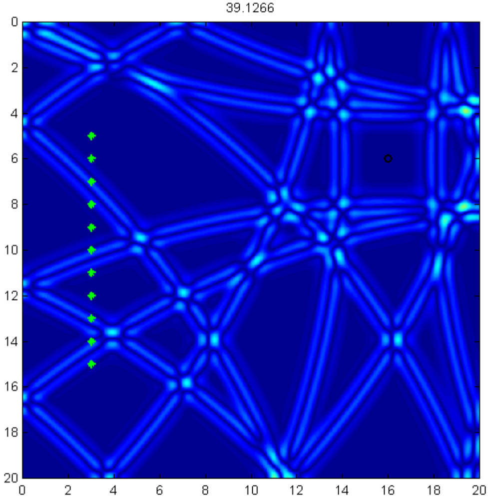


# Source localization - Forward step



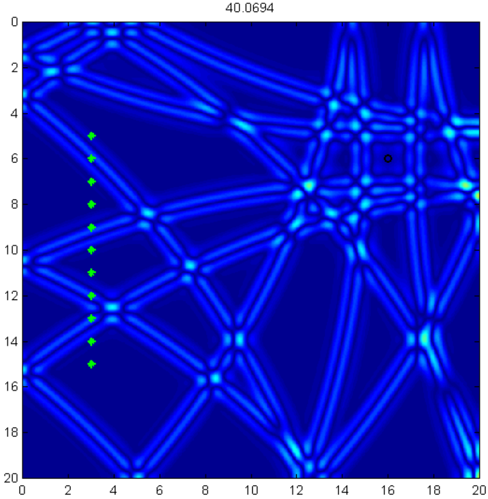

main

# Source localization - Forward step



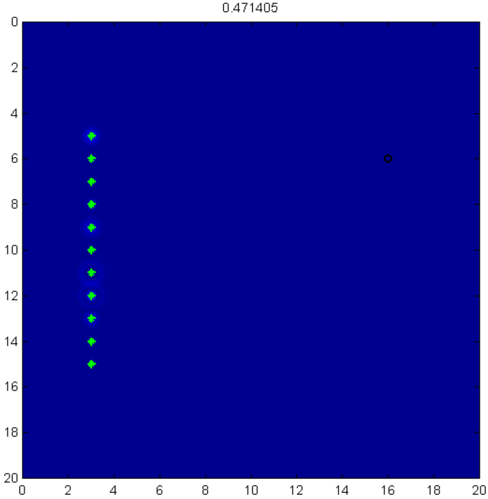

main

# Source localization - Forward step

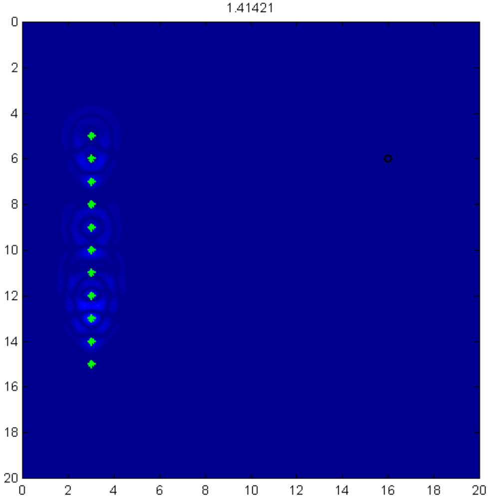



main

# Source localization - Backward step

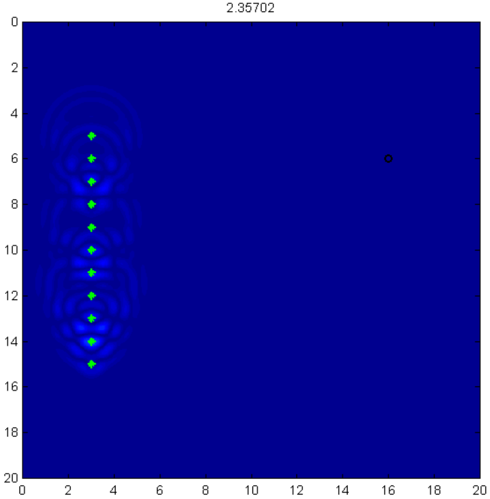


# Source localization - Backward step

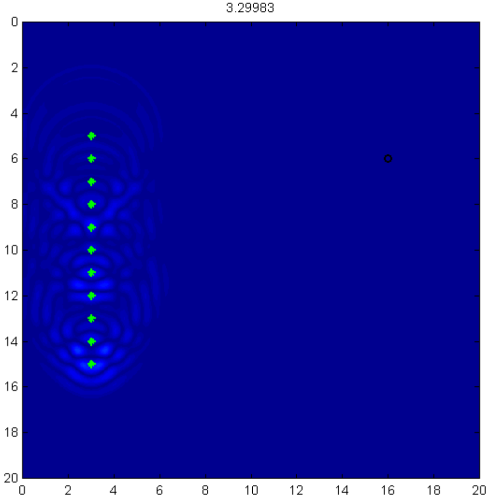



main

# Source localization - Backward step

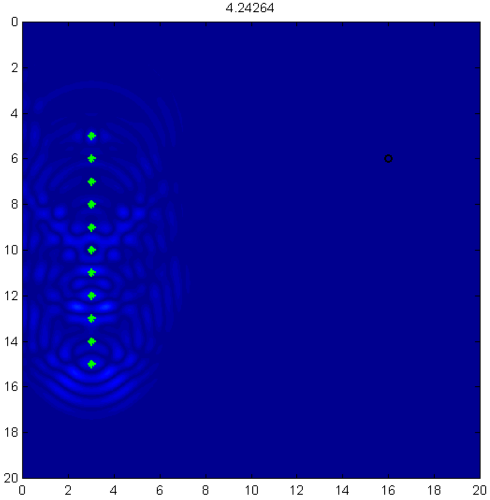


# Source localization - Backward step



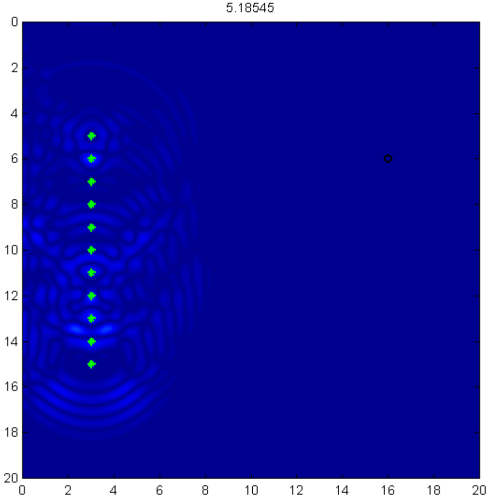

main

# Source localization - Backward step

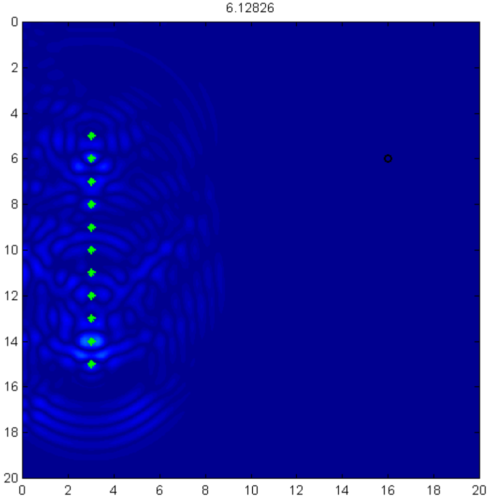



main

# Source localization - Backward step

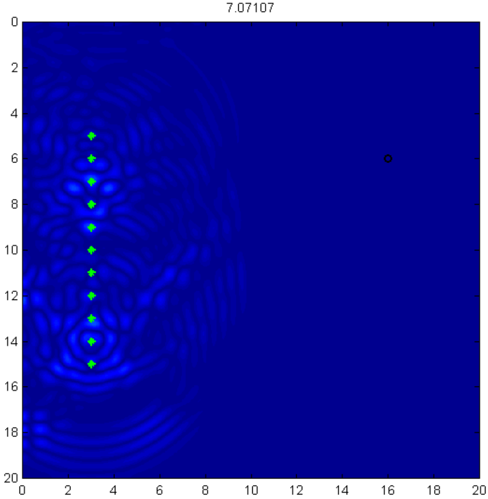


# Source localization - Backward step

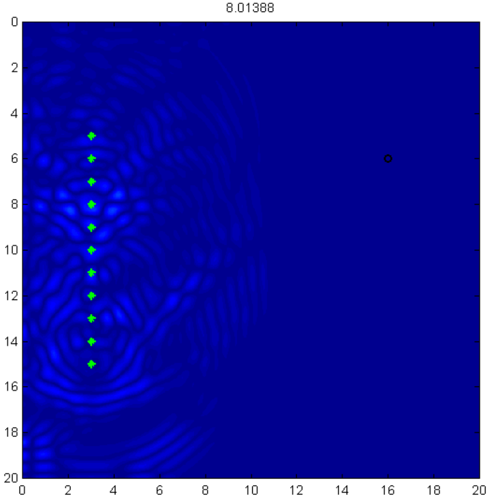



main

# Source localization - Backward step



# Source localization - Backward step



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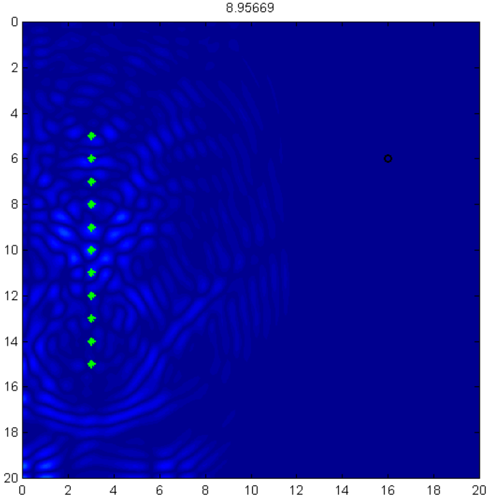
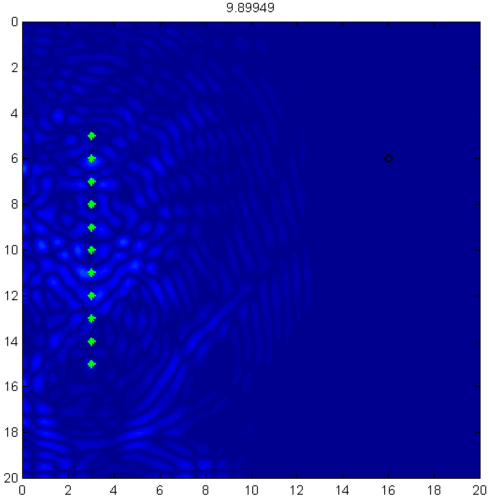
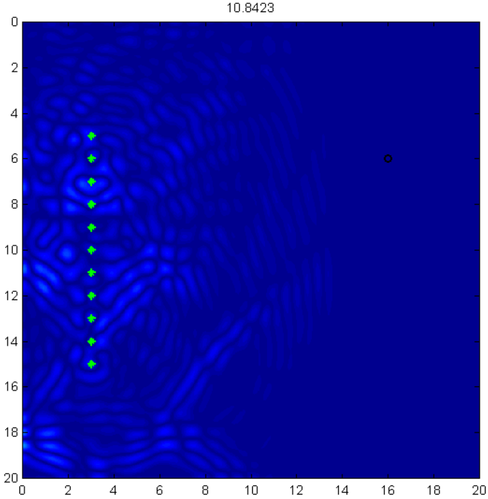



main

# Source localization - Backward step

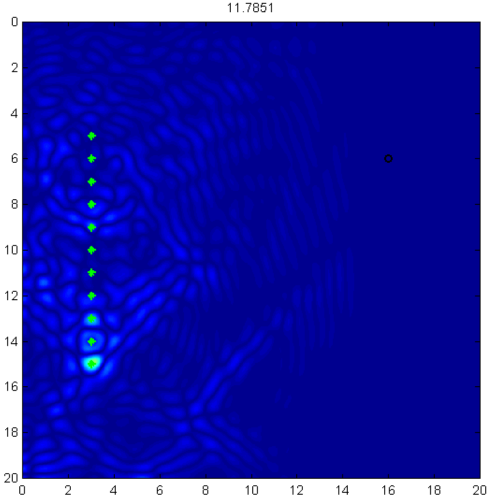


# Source localization - Backward step



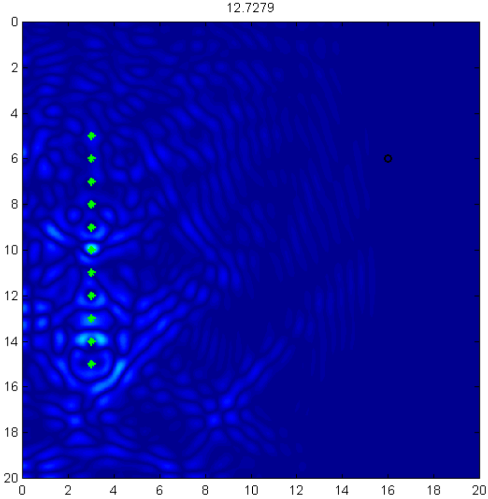

main

# Source localization - Backward step



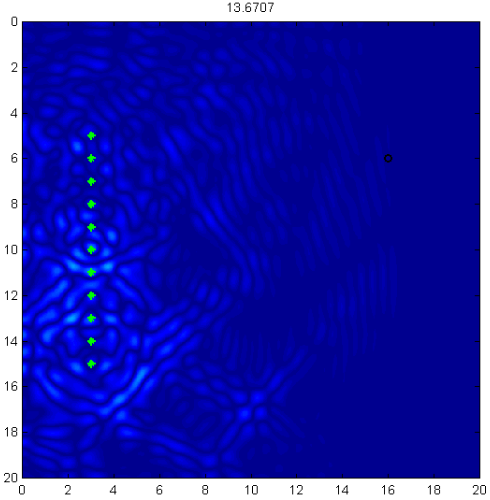

main

# Source localization - Backward step

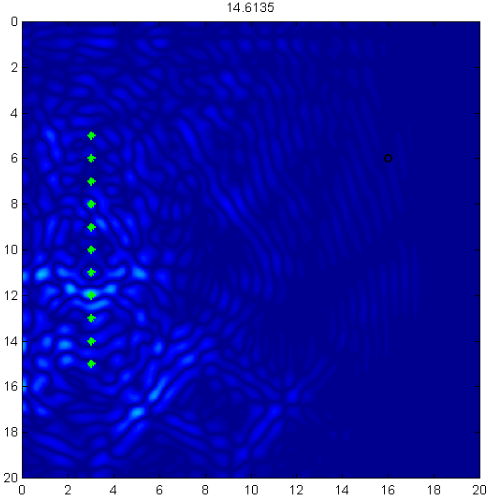



main

# Source localization - Backward step

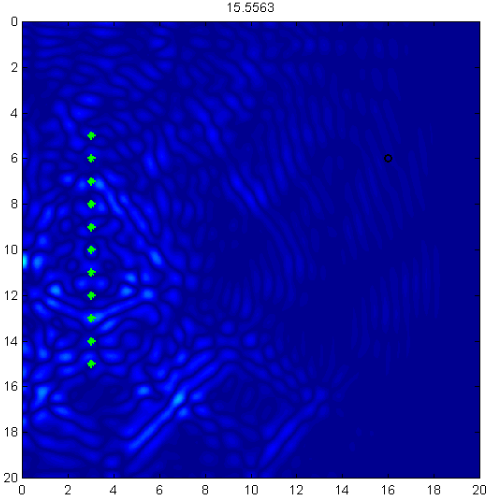


# Source localization - Backward step

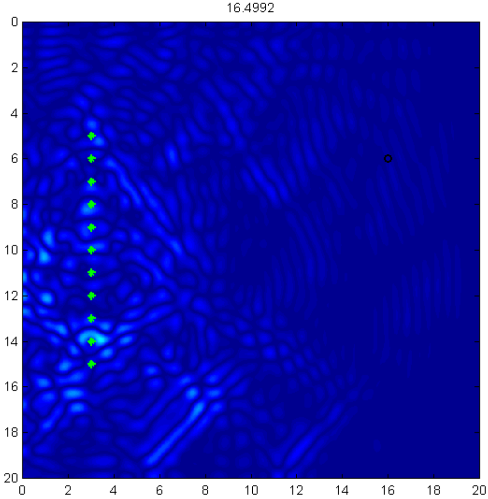



main

# Source localization - Backward step

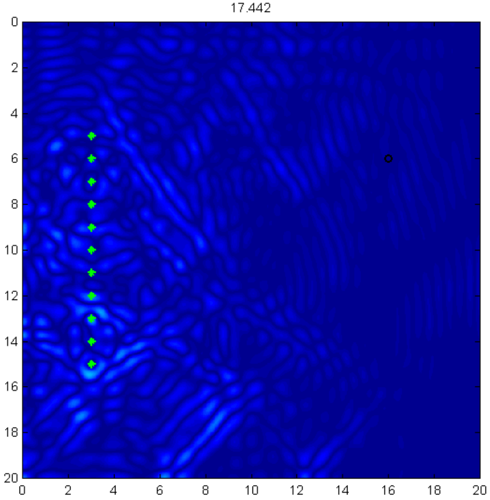


# Source localization - Backward step




main

# Source localization - Backward step



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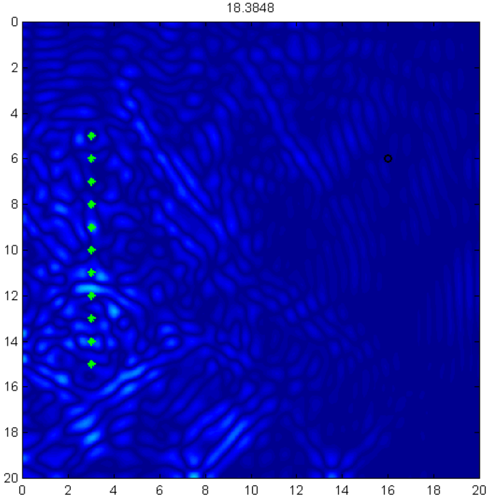

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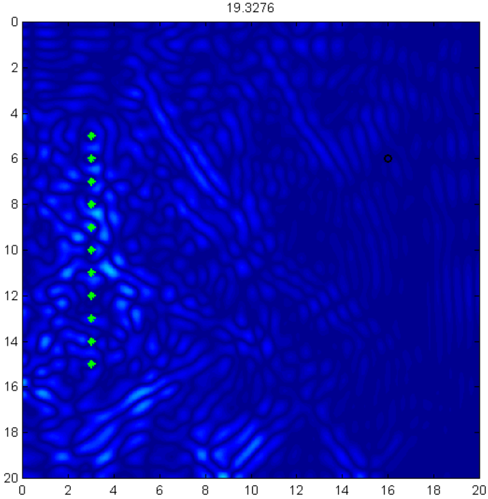
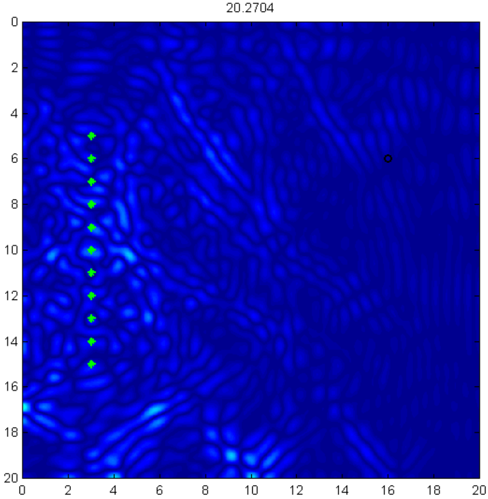



main

# Source localization - Backward step




main

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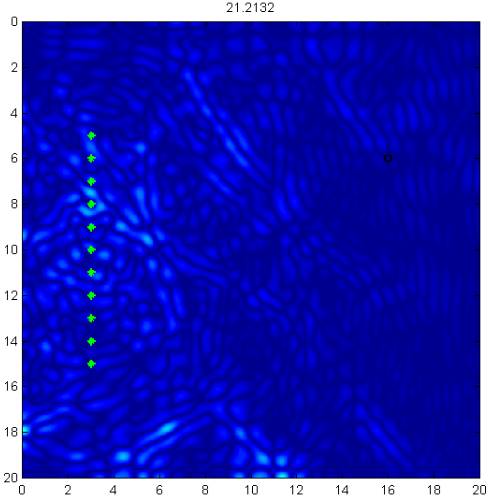

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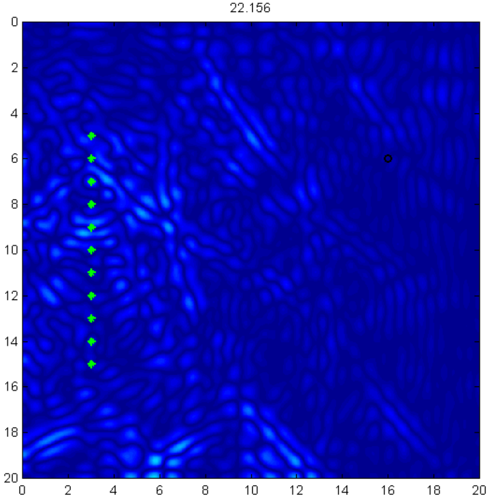

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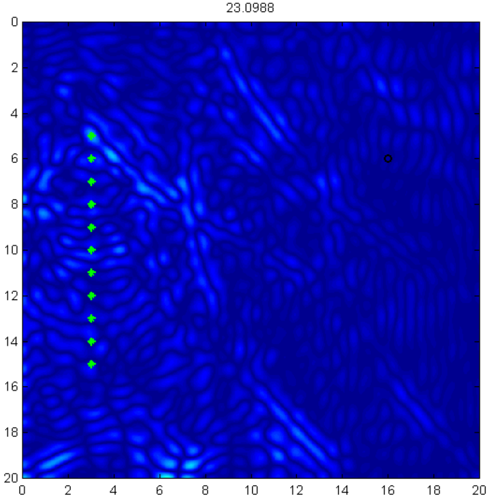


main

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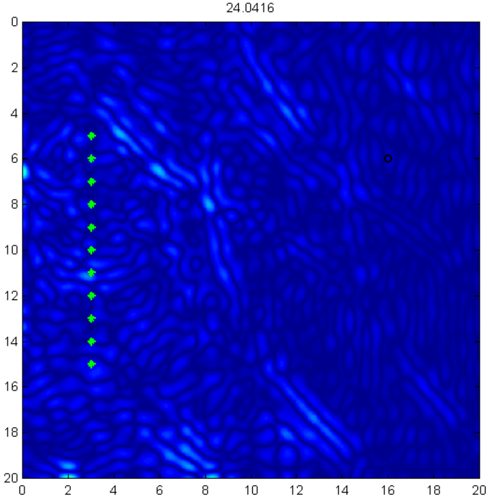


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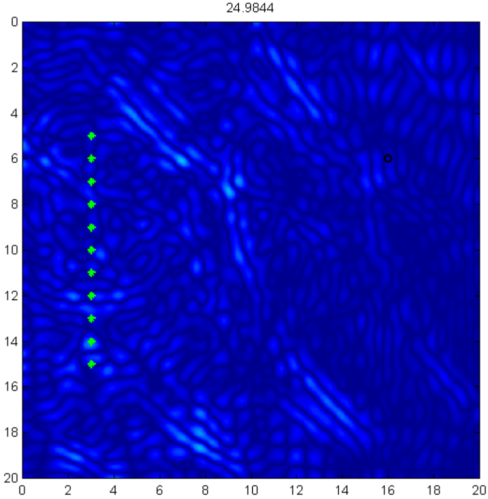

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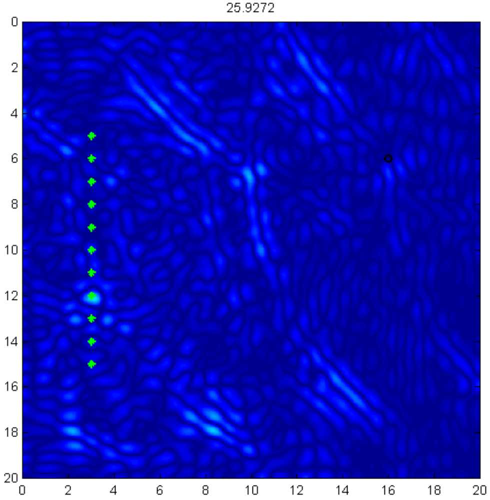
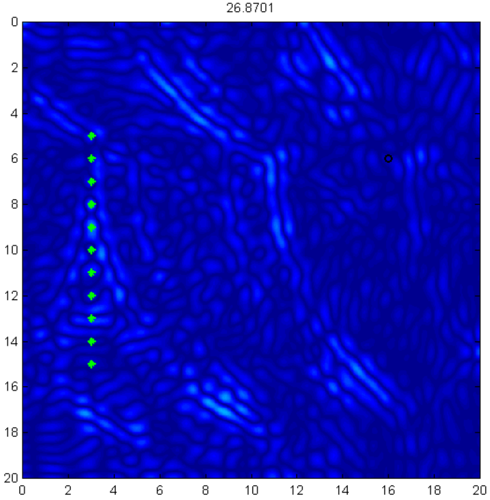



main

# Source localization - Backward step



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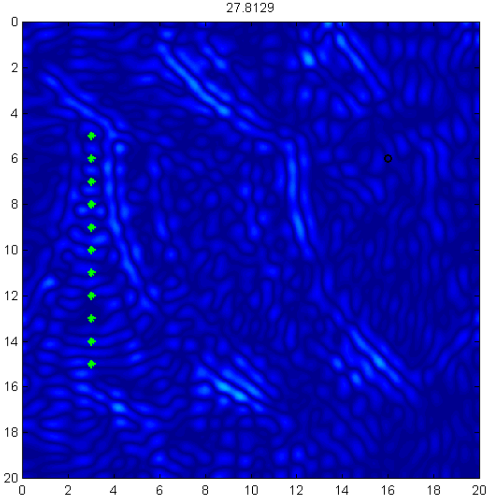


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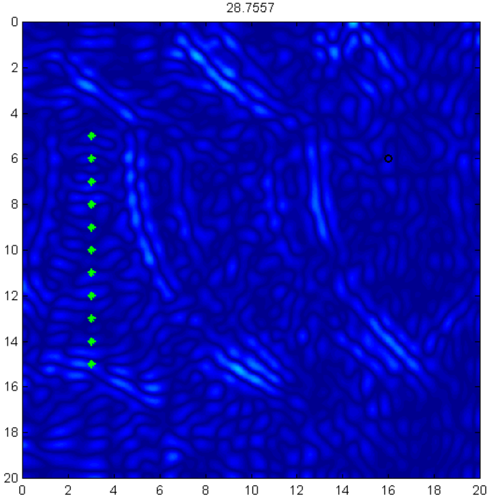
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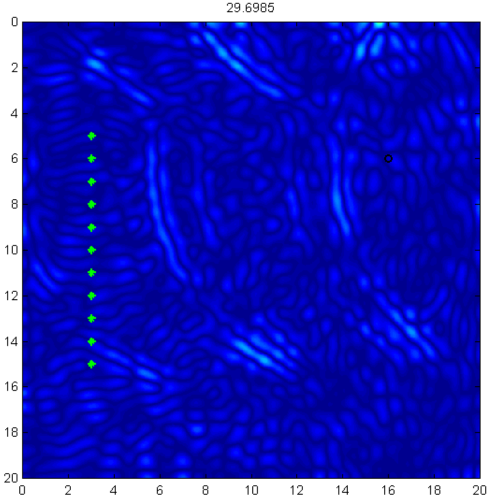


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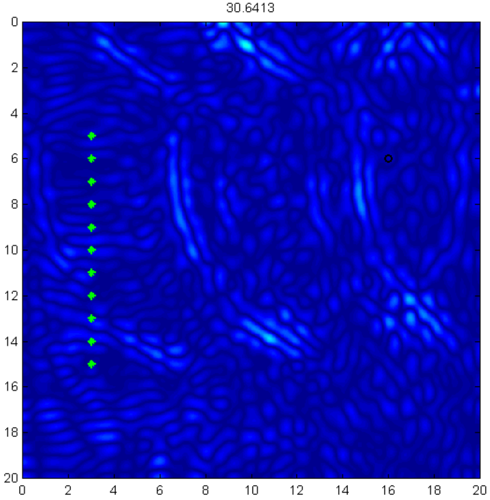

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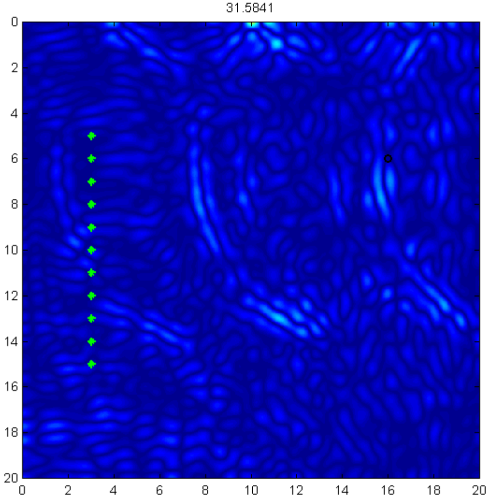


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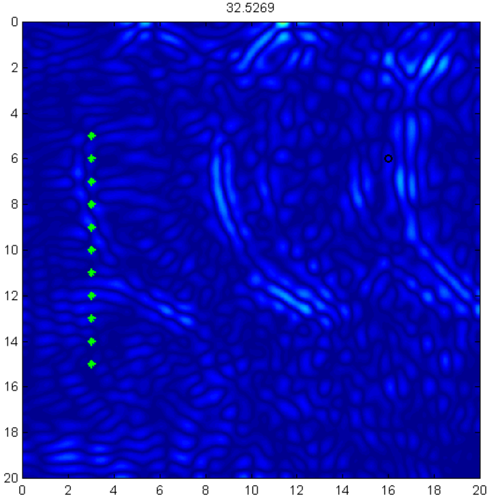

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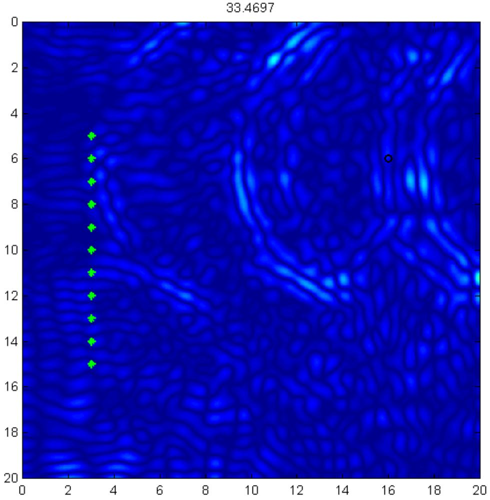

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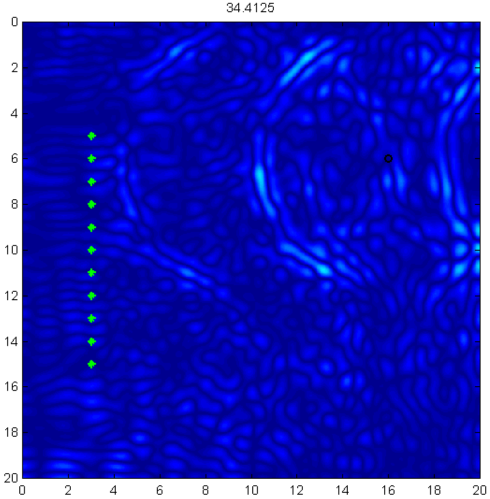
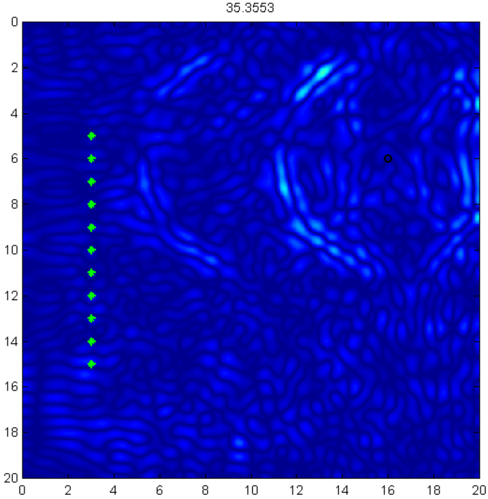
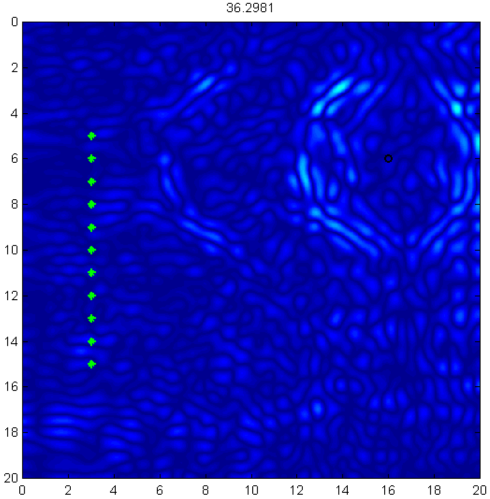



main

# Source localization - Backward step



# Source localization - Backward step

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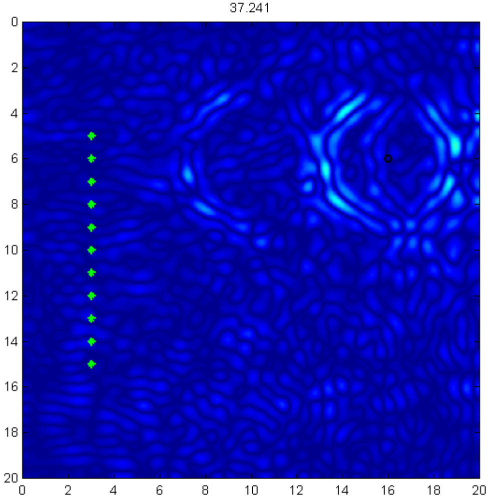


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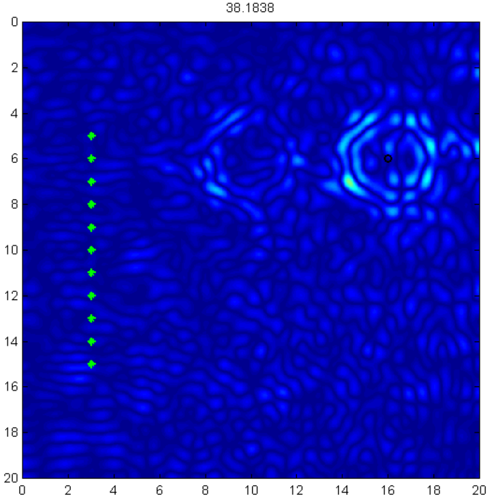


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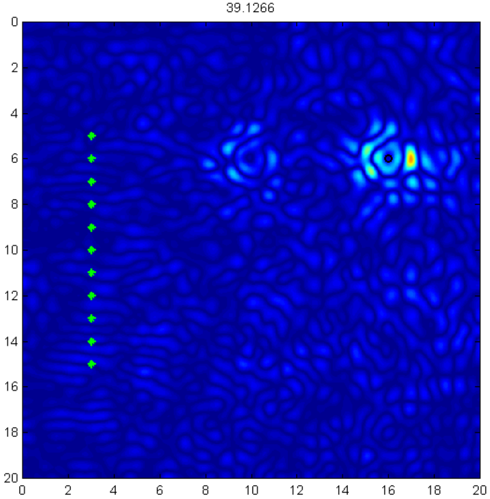
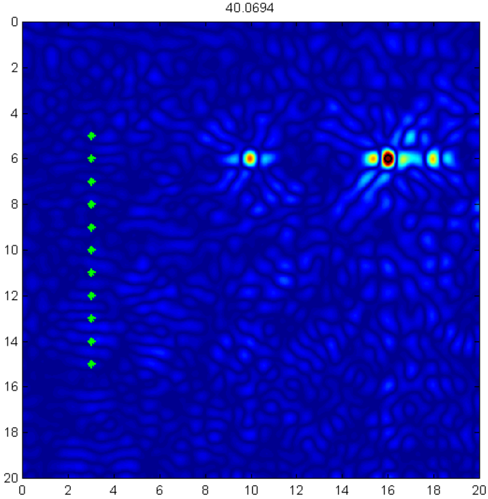


# Source localization - Backward step




main